Practice
Practice

Relations and Functions

State the domain and range of each relation. Then state whether the relation is a function. Write yes or no.

1. \{(-1, 2), (3, 10), (-2, 20), (3, 11)\}  2. \{(0, 2), (13, 6), (2, 2), (3, 1)\}

3. \{(1, 4), (2, 8), (3, 24)\}  4. \{(-1, -2), (3, 54), (-2, -16), (3, 81)\}

5. The domain of a relation is all even negative integers greater than -9. The range \(y\) of the relation is the set formed by adding 4 to the numbers in the domain. Write the relation as a table of values and as an equation. Then graph the relation.

Evaluate each function for the given value.

6. \(f(-2)\) if \(f(x) = 4x^3 + 6x^2 + 3x\)  7. \(f(3)\) if \(f(x) = 5x^2 - 4x - 6\)

8. \(h(t)\) if \(h(x) = 9x^9 - 4x^4 + 3x - 2\)  9. \(f(g + 1)\) if \(f(x) = x^2 - 2x + 1\)

10. Climate  The table shows record high and low temperatures for selected states.
   a. State the relation of the data as a set of ordered pairs.

   b. State the domain and range of the relation.

   c. Determine whether the relation is a function.

<table>
<thead>
<tr>
<th>State</th>
<th>High</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alabama</td>
<td>112</td>
<td>-27</td>
</tr>
<tr>
<td>Delaware</td>
<td>110</td>
<td>-17</td>
</tr>
<tr>
<td>Idaho</td>
<td>118</td>
<td>-60</td>
</tr>
<tr>
<td>Michigan</td>
<td>112</td>
<td>-51</td>
</tr>
<tr>
<td>New Mexico</td>
<td>122</td>
<td>-50</td>
</tr>
<tr>
<td>Wisconsin</td>
<td>114</td>
<td>-54</td>
</tr>
</tbody>
</table>

Source: National Climatic Data Center
Composition of Functions

Given \( f(x) = 2x^2 + 8 \) and \( g(x) = 5x - 6 \), find each function.

1. \((f + g)(x)\)
2. \((f - g)(x)\)

3. \((f \cdot g)(x)\)
4. \((\frac{f}{g})(x)\)

Find \([f \circ g](x)\) and \([g \circ f](x)\) for each \(f(x)\) and \(g(x)\).

5. \(f(x) = x + 5\)  
   \(g(x) = x - 3\)
6. \(f(x) = 2x^3 - 3x^2 + 1\)  
   \(g(x) = 3x\)

7. \(f(x) = 2x^2 - 5x + 1\)  
   \(g(x) = 2x - 3\)
8. \(f(x) = 3x^2 - 2x + 5\)  
   \(g(x) = 2x - 1\)

9. State the domain of \([f \circ g](x)\) for \(f(x) = \sqrt{x - 2}\) and \(g(x) = 3x\).

Find the first three iterates of each function using the given initial value.

10. \(f(x) = 2x - 6; x_0 = 1\)
11. \(f(x) = x^2 - 1; x_0 = 2\)

12. Fitness  Tara has decided to start a walking program. Her initial walking time is 5 minutes. She plans to double her walking time and add 1 minute every 5 days. Provided that Tara achieves her goal, how many minutes will she be walking on days 21 through 25?
Graphing Linear Equations

Graph each equation using the x- and y-intercepts.
1. \(2x - y - 6 = 0\)
2. \(4x + 2y + 8 = 0\)

Graph each equation using the y-intercept and the slope.
3. \(y = 5x - \frac{1}{2}\)
4. \(y = \frac{1}{2}x\)

Find the zero of each function. Then graph the function.
5. \(f(x) = 4x - 3\)
6. \(f(x) = 2x + 4\)

7. Business In 1990, a two-bedroom apartment at Remington Square Apartments rented for $575 per month. In 1999, the same two-bedroom apartment rented for $850 per month. Assuming a constant rate of increase, what will a tenant pay for a two-bedroom apartment at Remington Square in the year 2000?
Writing Linear Equations

Write an equation in slope-intercept form for each line described.

1. slope = −4, y-intercept = 3
2. slope = 5, passes through $A(-3, 2)$

3. slope = −4, passes through $B(3, 8)$
4. slope = $\frac{4}{3}$, passes through $C(-9, 4)$

5. slope = 1, passes through $D(-6, 6)$
6. slope = −1, passes through $E(3, -3)$

7. slope = 3, y-intercept = $\frac{3}{4}$
8. slope = −2, y-intercept = −7

9. slope = −1, passes through $F(-1, 7)$
10. slope = 0, passes through $G(3, 2)$

11. **Aviation** The number of active certified commercial pilots has been declining since 1980, as shown in the table.
   
   a. Find a linear equation that can be used as a model to predict the number of active certified commercial pilots for any year. Assume a steady rate of decline.

   b. Use the model to predict the number of pilots in the year 2003.

<table>
<thead>
<tr>
<th>Year</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>182,097</td>
</tr>
<tr>
<td>1985</td>
<td>155,929</td>
</tr>
<tr>
<td>1990</td>
<td>149,666</td>
</tr>
<tr>
<td>1993</td>
<td>143,014</td>
</tr>
<tr>
<td>1994</td>
<td>138,728</td>
</tr>
<tr>
<td>1995</td>
<td>133,980</td>
</tr>
<tr>
<td>1996</td>
<td>129,187</td>
</tr>
</tbody>
</table>

Source: U. S. Dept. of Transportation
Practice

Writing Equations of Parallel and Perpendicular Lines

Determine whether the graphs of each pair of equations are parallel, perpendicular, coinciding, or none of these.

1. \( \frac{x}{3} + 3y = 18 \)  
   \( 3x + 9y = 12 \)

2. \( 2x - 4y = 8 \)  
   \( x - 2y = 4 \)

3. \( -3x + 2y = 6 \)  
   \( 2x + 3y = 12 \)

4. \( x + y = 6 \)  
   \( 3x - y = 6 \)

5. \( 4x + 8y = 2 \)  
   \( 2x + 4y = 8 \)

6. \( 3x - y = 9 \)  
   \( 6x - 2y = 18 \)

Write the standard form of the equation of the line that is parallel to the graph of the given equation and that passes through the point with the given coordinates.

7. \( 2x + y - 5 = 0; (0, 4) \)

8. \( 3x - y + 3 = 0; (-1, -2) \)

9. \( 3x - 2y + 8 = 0; (2, 5) \)

Write the standard form of the equation of the line that is perpendicular to the graph of the given equation and that passes through the point with the given coordinates.

10. \( 2x - y + 6 = 0; (0, -3) \)

11. \( 2x - 5y - 6 = 0; (-4, 2) \)

12. \( 3x + 4y - 13 = 0; (2, 7) \)

13. **Consumerism**  Marillia paid $180 for 3 video games and 4 books. Three months later she purchased 8 books and 6 video games. Her brother guessed that she spent $320. Assuming that the prices of video games and books did not change, is it possible that she spent $320 for the second set of purchases? Explain.
Practice
Modeling Real-World Data with Linear Functions

Complete the following for each set of data.

a. Graph the data on a scatter plot.
b. Use two ordered pairs to write the equation of a best-fit line.
c. If the equation of the regression line shows a moderate or strong relationship, predict the missing value. Explain whether you think the prediction is reliable.

1. U.S. Life Expectancy

<table>
<thead>
<tr>
<th>Birth Year</th>
<th>Number of Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>75.4</td>
</tr>
<tr>
<td>1991</td>
<td>75.5</td>
</tr>
<tr>
<td>1992</td>
<td>75.8</td>
</tr>
<tr>
<td>1993</td>
<td>75.5</td>
</tr>
<tr>
<td>1994</td>
<td>75.7</td>
</tr>
<tr>
<td>1995</td>
<td>75.8</td>
</tr>
<tr>
<td>2015</td>
<td>?</td>
</tr>
</tbody>
</table>

Source: National Center for Health Statistics

2. Population Growth

<table>
<thead>
<tr>
<th>Year</th>
<th>Population (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>252.1</td>
</tr>
<tr>
<td>1992</td>
<td>255.0</td>
</tr>
<tr>
<td>1993</td>
<td>257.7</td>
</tr>
<tr>
<td>1994</td>
<td>260.3</td>
</tr>
<tr>
<td>1995</td>
<td>262.8</td>
</tr>
<tr>
<td>1996</td>
<td>265.2</td>
</tr>
<tr>
<td>1997</td>
<td>267.7</td>
</tr>
<tr>
<td>1998</td>
<td>270.3</td>
</tr>
<tr>
<td>1999</td>
<td>272.9</td>
</tr>
<tr>
<td>2010</td>
<td>?</td>
</tr>
</tbody>
</table>

Source: U.S. Census Bureau
Graph each function.

1. \( f(x) = \begin{cases} 
1 & \text{if } x \geq 2 \\
3x - 2 & \text{if } -1 \leq x < 2 \\
x & \text{if } x < -2 
\end{cases} \)

2. \( f(x) = \begin{cases} 
-2 & \text{if } x \leq -1 \\
1 + x & \text{if } -1 < x < 2 \\
1 - x & \text{if } x > 2 
\end{cases} \)

3. \( f(x) = |x| - 3 \)

4. \( f(x) = [x] - 1 \)

5. \( f(x) = 3|x| - 2 \)

6. \( f(x) = [2x + 1] \)

7. Graph the tax rates for the different incomes by using a step function.

<table>
<thead>
<tr>
<th>Limits of Taxable Income</th>
<th>Tax Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 to $41,200</td>
<td>15%</td>
</tr>
<tr>
<td>$41,201 to $99,600</td>
<td>28%</td>
</tr>
<tr>
<td>$99,601 to $151,750</td>
<td>31%</td>
</tr>
<tr>
<td>$151,751 to $271,050</td>
<td>36%</td>
</tr>
<tr>
<td>$271,051 and up</td>
<td>39.6%</td>
</tr>
</tbody>
</table>

Source: Information Please Almanac
Practice

Graphing Linear Inequalities

Graph each inequality.

1. $x \geq -2$

2. $y < -2x - 4$

3. $y \geq 3x + 2$

4. $y < |x + 3|$

5. $y > |x - 2|$

6. $y \leq -\frac{1}{2}x + 4$

7. $\frac{3}{4}x - 3 \leq y \leq \frac{4}{5}x + 4$

8. $-4 \leq x - 2y < 6$
Practice

Solving Systems of Equations in Two Variables

State whether each system is consistent and independent, consistent and dependent, or inconsistent.

1. \(-x + y = -4\)  
   \(3x - 3y = 12\)

2. \(2x - 5y = 8\)  
   \(15y - 6x = -24\)

Solve each system of equations by graphing.

3. \(x + y = 6\)  
   \(2x + 3y = 12\)

4. \(x + y = 6\)  
   \(3x - y = 6\)

Solve each system of equations algebraically.

5. \(x + y = 4\)  
   \(3x - 2y = 7\)

6. \(3x - 4y = 10\)  
   \(-3x + 4y = 8\)

7. \(4x - 3y = 15\)  
   \(2x + y = 5\)

8. \(4x + 5y = 11\)  
   \(3x - 2y = -9\)

9. \(2x + 3y = 19\)  
   \(7x - y = 9\)

10. \(2x - y = 6\)  
    \(x + y = 6\)

11. **Real Estate**  
    AMC Homes, Inc. is planning to build three- and four-bedroom homes in a housing development called Chestnut Hills. Consumer demand indicates a need for three times as many four-bedroom homes as for three-bedroom homes. The net profit from each three-bedroom home is $16,000 and from each four-bedroom home, $17,000. If AMC Homes must net a total profit of $13.4 million from this development, how many homes of each type should they build?
Practice

Solving Systems of Equations in Three Variables

Solve each system of equations.

1. \(x + y - z = -1\)
   \(x + y + z = 3\)
   \(3x - 2y - z = -4\)

2. \(x + y = 5\)
   \(3x + z = 2\)
   \(4y - z = 8\)

3. \(3x - 5y + z = 8\)
   \(4y - z = 10\)
   \(7x + y = 4\)

4. \(2x + 3y + 3z = 2\)
   \(10x - 6y + 3z = 0\)
   \(4x - 3y - 6z = 2\)

5. \(2x - y + z = -1\)
   \(x - y + z = 1\)
   \(x - 2y + z = 2\)

6. \(4x + 4y - 2z = 3\)
   \(-6x - 6y + 6z = 5\)
   \(2x - 3y - 4z = 2\)

7. \(x - z = 5\)
   \(y + 3z = 12\)
   \(2x + y = 7\)

8. \(2x + 4y - 2z = 9\)
   \(4x - 6y + 2z = -9\)
   \(x - y + 3z = 4\)

9. **Business**  The president of Speedy Airlines has discovered that her competitor, Zip Airlines, has purchased 13 new airplanes from Commuter Aviation for a total of $15.9 million. She knows that Commuter Aviation produces three types of planes and that type A sells for $1.1 million, type B sells for $1.2 million, and type C sells for $1.7 million. The president of Speedy Airlines also managed to find out that Zip Airlines purchased 5 more type A planes than type C planes. How many planes of each type did Zip Airlines purchase?
Practice

Modeling Real-World Data with Matrices

Find the values of \( x \) and \( y \) for which each matrix equation is true.

1. \[
\begin{bmatrix}
x \\
y
\end{bmatrix}
= \begin{bmatrix}
2y - 4 \\
2x
\end{bmatrix}
\]

2. \[
\begin{bmatrix}
2x - 3 \\
4y
\end{bmatrix}
= \begin{bmatrix}
y \\
3x
\end{bmatrix}
\]

Use matrices \( A, B, \) and \( C \) to find each sum, difference, or product.

\[
A = \begin{bmatrix}
-1 & 5 & 6 \\
2 & -7 & -2 \\
4 & 4 & 2
\end{bmatrix}
\quad B = \begin{bmatrix}
2 & 3 & 1 \\
-1 & 1 & 4 \\
5 & -2 & 3
\end{bmatrix}
\quad C = \begin{bmatrix}
8 & 10 & -9 \\
-6 & 12 & 14
\end{bmatrix}
\]

3. \( A + B \)

4. \( A - B \)

5. \( B - A \)

6. \(-2A\)

7. \( CA \)

8. \( AB \)

9. \( AA \)

10. \( CB \)

11. \((CA)B\)

12. \(C(AB)\)

13. **Entertainment** On one weekend, the Goxfield Theater reported the following ticket sales for three first-run movies, as shown in the matrix at the right. If the ticket prices were $6 for each adult and $4 for each child, what were the weekend sales for each movie.

\[
\begin{bmatrix}
1021 & 523 \\
2547 & 785 \\
3652 & 2456
\end{bmatrix}
\]

Adults  |  Children
---------|-----------
Movie 1  | 1021      | 523      |
Movie 2  | 2547      | 785      |
Movie 3  | 3652      | 2456     |
**Practice**

**Modeling Motion with Matrices**

*Use scalar multiplication to determine the coordinates of the vertices of each dilated figure. Then graph the pre-image and the image on the same coordinate grid.*

1. triangle with vertices $A(1, 2), B(2, -1),$ and $C(-2, 0);$ scale factor 2

2. quadrilateral with vertices $E(-2, -7), F(4, -3), G(0, 1),$ and $H(-4, -2);$ scale factor 0.5

*Use matrices to determine the coordinates of the vertices of each translated figure. Then graph the pre-image and the image on the same coordinate grid.*

3. square with vertices $W(1, -3), X(-4, -2),$ $Y(-3, 3),$ and $Z(2, 2)$ translated 2 units right and 3 units down

4. triangle with vertices $J(3, 1), K(2, -4),$ and $L(0, -2)$ translated 4 units left and 2 units up

*Use matrices to determine the coordinates of the vertices of each reflected figure. Then graph the pre-image and the image on the same coordinate grid.*

5. $\triangle MNP$ with vertices $M(-3, 4), N(3, 1),$ and $P(-4, -3)$ reflected over the $y$-axis

6. a rhombus with vertices $Q(2, 3),$ $R(4, -1), S(-1, -2),$ and $T(-3, 2)$ reflected over the line $y = x$

*Use matrices to determine the coordinates of the vertices of each rotated figure. Then graph the pre-image and the image on the same coordinate grid.*

7. quadrilateral $CDFG$ with vertices $C(-2, 3),$ $D(3, 4), F(3, -1),$ and $G(-3, -4)$ rotated $90^\circ$

8. Pentagon $VWXYZ$ with vertices $V(1, 3),$ $W(4, 2), X(3, -2),$ $Y(-1, -4), Z(-2, 1)$ rotated $180^\circ$
Practice

Determinants and Multiplicative Inverses of Matrices

Find the value of each determinant.
1. \[
\begin{vmatrix}
-2 & 3 \\
8 & -12
\end{vmatrix}
\]
2. \[
\begin{vmatrix}
3 & -5 \\
7 & 9
\end{vmatrix}
\]
3. \[
\begin{vmatrix}
1 & -1 & 0 \\
2 & 1 & 4 \\
5 & -3 & 5
\end{vmatrix}
\]
4. \[
\begin{vmatrix}
2 & 3 & 1 \\
-3 & -1 & 5 \\
1 & -4 & 2
\end{vmatrix}
\]

Find the inverse of each matrix, if it exists.
5. \[
\begin{vmatrix}
3 & 8 \\
-1 & 5
\end{vmatrix}
\]
6. \[
\begin{vmatrix}
5 & 2 \\
10 & 4
\end{vmatrix}
\]

Solve each system by using matrix equations.
7. \[
\begin{align*}
2x - 3y &= 17 \\
3x + y &= 9
\end{align*}
\]
8. \[
\begin{align*}
4x - 3y &= -16 \\
2x + 5y &= 18
\end{align*}
\]

Solve each matrix equation.
9. \[
\begin{bmatrix}
2 & -1 & 3 \\
1 & 2 & 1 \\
-1 & -3 & -2
\end{bmatrix}
\cdot
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} =
\begin{bmatrix}
-8 \\
3 \\
-7
\end{bmatrix}, \text{ if the inverse is }
\begin{bmatrix}
-1 & -11 & -7 \\
1 & -1 & 1 \\
6 & 7 & 5
\end{bmatrix}
\]

10. \[
\begin{bmatrix}
5 & -2 & 4 \\
3 & -4 & 2 \\
1 & -3 & 1
\end{bmatrix}
\cdot
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} =
\begin{bmatrix}
-2 \\
0 \\
1
\end{bmatrix}, \text{ if the inverse is }
\begin{bmatrix}
2 & -10 & 12 \\
-1 & 1 & 2 \\
-5 & 13 & -14
\end{bmatrix}
\]

11. **Landscaping**  Two dump truck have capacities of 10 tons and 12 tons. They make a total of 20 round trips to haul 226 tons of topsoil for a landscaping project. How many round trips does each truck make?
Solving Systems of Linear Inequalities

Solve each system of inequalities by graphing.

1. \(-4x + 7y \geq -21; 3x + 7y \leq 28\)

2. \(x \leq 3; y \leq 5; x + y \geq 1\)

Solve each system of inequalities by graphing. Name the coordinates of the vertices of the polygonal convex set.

3. \(x \geq 0; y \geq 0; y \geq x - 4; 7x + 6y \leq 54\)

4. \(x \geq 0; y + 2 \geq 0; 5x + 6y \leq 18\)

Find the maximum and minimum values of each function for the polygonal convex set determined by the given system of inequalities.

5. \(3x - 2y \geq 0 \quad y \geq 0\)  
   \(3x + 2y \leq 24 \quad f(x, y) = 7y - 3x\)

6. \(y \leq -x + 8 \quad 4x - 3y \geq -3\)  
   \(x + 8y \geq 8 \quad f(x, y) = 4x - 5y\)

7. Business  Henry Jackson, a recent college graduate, plans to start his own business manufacturing bicycle tires. Henry knows that his start-up costs are going to be $3000 and that each tire will cost him at least $2 to manufacture. In order to remain competitive, Henry cannot charge more than $5 per tire. Draw a graph to show when Henry will make a profit.
Graph each system of inequalities. In a problem asking you to find the maximum value of \( f(x, y) \), state whether the situation is infeasible, has alternate optimal solutions, or is unbounded. In each system, assume that \( x \geq 0 \) and \( y \geq 0 \) unless stated otherwise.

1. \(-2y \leq 2x - 36\)
   \( x + y \geq 30 \)
   \( f(x, y) = 3x + 3y \)

2. \(2x + 2y \geq 10\)
   \(2x + y \geq 8\)
   \( f(x, y) = x + y \)

Solve each problem, if possible. If not possible, state whether the problem is infeasible, has alternate optimal solutions, or is unbounded.

3. Nutrition
   A diet is to include at least 140 milligrams of Vitamin A and at least 145 milligrams of Vitamin B. These requirements can be obtained from two types of food. Type X contains 10 milligrams of Vitamin A and 20 milligrams of Vitamin B per pound. Type Y contains 30 milligrams of Vitamin A and 15 milligrams of Vitamin B per pound. If type X food costs $12 per pound and type Y food costs $8 per pound how many pounds of each type of food should be purchased to satisfy the requirements at the minimum cost?

4. Manufacturing
   The Cruiser Bicycle Company makes two styles of bicycles: the Traveler, which sells for $200, and the Tourister, which sells $600. Each bicycle has the same frame and tires, but the assembly and painting time required for the Traveler is only 1 hour, while it is 3 hours for the Tourister. There are 300 frames and 360 hours of labor available for production. How many bicycles of each model should be produced to maximize revenue?
Practice
Symmetry and Coordinate Graphs

Determine whether the graph of each function is symmetric with respect to the origin.

1. \( f(x) = \frac{-12}{x} \)  
2. \( f(x) = x^5 - 2 \)
3. \( f(x) = x^3 - 4x \)  
4. \( f(x) = \frac{x^2}{3-x} \)

Determine whether the graph of each equation is symmetric with respect to the x-axis, the y-axis, the line \( y = x \), the line \( y = -x \), or none of these.

5. \( x + y = 6 \)  
6. \( x^2 + y = 2 \)
7. \( xy = 3 \)  
8. \( x^3 + y^2 = 4 \)
9. \( y = 4x \)  
10. \( y = x^2 - 1 \)

11. Is \( f(x) = |x| \) an even function, an odd function, or neither?

Refer to the graph at the right for Exercises 12 and 13.

12. Complete the graph so that it is the graph of an odd function.

13. Complete the graph so that it is the graph of an even function.

14. Geometry Cameron told her friend Juanita that the graph of \( |y| = 6 - |3x| \) has the shape of a geometric figure. Determine whether the graph of \( |y| = 6 - |3x| \) is symmetric with respect to the x-axis, the y-axis, both, or neither Then make a sketch of the graph. Is Cameron correct?
Describe how the graphs of \( f(x) \) and \( g(x) \) are related.

1. \( f(x) = x^2 \) and \( g(x) = (x + 3)^2 - 1 \)
2. \( f(x) = |x| \) and \( g(x) = -|2x| \)

Use the graph of the given parent function to describe the graph of each related function.

3. \( f(x) = x^3 \)
   a. \( y = 2x^3 \)
   b. \( y = -0.5(x - 2)^3 \)
   c. \( y = |(x + 1)^3| \)
4. \( f(x) = \sqrt{x} \)
   a. \( y = \sqrt{x + 3} + 1 \)
   b. \( y = \sqrt{-x} - 2 \)
   c. \( y = \sqrt{0.25x - 4} \)

Sketch the graph of each function.

5. \( f(x) = -(x - 1)^2 + 1 \)
6. \( f(x) = 2|x + 2| - 3 \)

7. Consumer Costs  During her free time, Jill baby-sits the neighborhood children. She charges $4.50 for each whole hour or any fraction of an hour. Write and graph a function that shows the cost of \( x \) hours of baby-sitting.
Practice

Graphs of Nonlinear Inequalities

Determine whether the ordered pair is a solution for the given inequality.
Write yes or no.

1. \( y > (x + 2)^2 + 3, (-2, 6) \)
2. \( y < (x - 3)^3 + 2, (4, 5) \)
3. \( y \leq |2x - 4| - 1, (-4, 1) \)

Graph each inequality.

4. \( y \leq 2|x - 1| \)

5. \( y > 2(x - 1)^2 \)

6. \( y < \sqrt{x - 2} + 1 \)

7. \( y \geq (x + 3)^3 \)

Solve each inequality.

8. \( |4x - 10| \leq 6 \)

9. \( |x + 5| + 2 > 6 \)

10. \( |2x - 2| - 1 < 7 \)

11. Measurement  Instructions for building a birdhouse warn that the platform, which ideally measures 14.75 cm², should not vary in size by more than 0.30 cm². If it does, the preconstructed roof for the birdhouse will not fit properly.
   a. Write an absolute value inequality that represents the range of possible sizes for the platform. Then solve for \( x \) to find the range.

   b. Dena cut a board 14.42 cm². Does the platform that Dena cut fit within the acceptable range?
Inverse Functions and Relations

Graph each function and its inverse.

1. \( f(x) = (x - 1)^3 + 1 \)
2. \( f(x) = 3|x| + 2 \)

\[ \text{Graphs of } f(x) \text{ and } f(x) \text{ inv} \]

Find \( f^{-1}(x) \). Then state whether \( f^{-1}(x) \) is a function.

3. \( f(x) = -4x^2 + 1 \)
4. \( f(x) = \sqrt[3]{x - 1} \)
5. \( f(x) = \frac{4}{(x - 3)^2} \)

Graph each equation using the graph of the given parent function.

6. \( y = -\sqrt{x + 3} - 1 \), \( p(x) = x^2 \)
7. \( y = 2 + \sqrt[5]{x + 2} \), \( p(x) = x^5 \)

8. **Fire Fighting**  Airplanes are often used to drop water on forest fires in an effort to stop the spread of the fire. The time \( t \) it takes the water to travel from height \( h \) to the ground can be derived from the equation \( h = \frac{1}{2}gt^2 \) where \( g \) is the acceleration due to gravity (32 feet/second\(^2\)).

   a. Write an equation that will give time as a function of height.

   b. Suppose a plane drops water from a height of 1024 feet. How many seconds will it take for the water to hit the ground?
Continuity and End Behavior

Determine whether each function is continuous at the given x-value. Justify your answer using the continuity test.

1. \( y = \frac{-2}{3x^2}; x = -1 \)

2. \( y = \frac{x^2 + x + 4}{2}; x = 1 \)

3. \( y = x^3 - 2x + 2; x = 1 \)

4. \( y = \frac{x - 2}{x + 4}; x = -4 \)

Describe the end behavior of each function.

5. \( y = 2x^5 - 4x \)

6. \( y = -2x^6 + 4x^4 - 2x + 1 \)

7. \( y = x^4 - 2x^3 + x \)

8. \( y = -4x^3 + 5 \)

Given the graph of the function, determine the interval(s) for which the function is increasing and the interval(s) for which the function is decreasing.

9. [Graph Image]

10. Electronics  Ohm’s Law gives the relationship between resistance \( R \), voltage \( E \), and current \( I \) in a circuit as \( R = \frac{E}{I} \). If the voltage remains constant but the current keeps increasing in the circuit, what happens to the resistance?
Critical Points and Extrema

Locate the extrema for the graph of \( y = f(x) \). Name and classify the extrema of the function.

1. 
2. 
3. 
4. 

Determine whether the given critical point is the location of a maximum, a minimum, or a point of inflection.

5. \( y = x^2 - 6x + 1, \ x = 3 \)  
6. \( y = x^2 - 2x - 6, \ x = 1 \)  
7. \( y = x^4 + 3x^2 - 5, \ x = 0 \)

8. \( y = x^5 - 2x^3 - 2x^2, \ x = 0 \)  
9. \( y = x^3 + x^2 - x, \ x = -1 \)  
10. \( y = 2x^3 + 4, \ x = 0 \)

11. **Physics** Suppose that during an experiment you launch a toy rocket straight upward from a height of 6 inches with an initial velocity of 32 feet per second. The height at any time \( t \) can be modeled by the function \( s(t) = -16t^2 + 32t + 0.5 \) where \( s(t) \) is measured in feet and \( t \) is measured in seconds. Graph the function to find the maximum height obtained by the rocket before it begins to fall.
Graphs of Rational Functions

Determine the equations of the vertical and horizontal asymptotes, if any, of each function.

1. \( f(x) = \frac{4}{x^2 + 1} \)
2. \( f(x) = \frac{2x + 1}{x + 1} \)
3. \( g(x) = \frac{x + 3}{(x + 1)(x - 2)} \)

Use the parent graph \( f(x) = \frac{1}{x} \) to graph each equation. Describe the transformation(s) that have taken place. Identify the new locations of the asymptotes.

4. \( y = \frac{3}{x + 1} - 2 \)
5. \( y = -\frac{4}{x - 3} + 3 \)

Determine the slant asymptotes of each equation.

6. \( y = \frac{5x^2 - 10x + 1}{x - 2} \)
7. \( y = \frac{x^2 - x}{x + 1} \)

8. Graph the function \( y = \frac{x^2 + x - 6}{x + 1} \).

9. Physics The illumination \( I \) from a light source is given by the formula \( I = \frac{k}{d^2} \), where \( k \) is a constant and \( d \) is distance. As the distance from the light source doubles, how does the illumination change?
Practice

Direct, Inverse, and Joint Variation
Write a statement of variation relating the variables of each equation.
Then name the constant of variation.

1. \( \frac{x^2}{y} = 3 \)

2. \( E = IR \)

3. \( y = 2x \)

4. \( d = 6t^2 \)

Find the constant of variation for each relation and use it to write an equation for each statement. Then solve the equation.

5. Suppose \( y \) varies directly as \( x \) and \( y = 35 \) when \( x = 5 \). Find \( y \) when \( x = 7 \).

6. If \( y \) varies directly as the cube of \( x \) and \( y = 3 \) when \( x = 2 \), find \( x \) when \( y = 24 \).

7. If \( y \) varies inversely as \( x \) and \( y = 3 \) when \( x = 25 \), find \( x \) when \( y = 10 \).

8. Suppose \( y \) varies jointly as \( x \) and \( z \), and \( y = 64 \) when \( x = 4 \) and \( z = 8 \).
   Find \( y \) when \( x = 7 \) and \( z = 11 \).

9. Suppose \( V \) varies jointly as \( h \) and the square of \( r \), and \( V = 45\pi \) when \( r = 3 \) and \( h = 5 \). Find \( r \) when \( V = 175\pi \) and \( h = 7 \).

10. If \( y \) varies directly as \( x \) and inversely as the square of \( z \), and \( y = -5 \) when \( x = 10 \) and \( z = 2 \), find \( y \) when \( x = 5 \) and \( z = 5 \).

11. **Finances** Enrique deposited $200.00 into a savings account. The simple interest \( I \) on his account varies jointly as the time \( t \) in years and the principal \( P \).
    After one quarter (three months), the interest on Enrique’s account is $2.75.
    Write an equation relating interest, principal, and time. Find the constant of variation. Then find the interest after three quarters.
Practice

Polynomial Functions

State the degree and leading coefficient of each polynomial.
1. \(6a^4 + a^3 - 2a\)
   \(\text{Degree: } 4, \text{ Leading Coefficient: } 6\)

2. \(3p^3 - 7p^5 - 2p^3 + 5\)
   \(\text{Degree: } 5, \text{ Leading Coefficient: } -7\)

Write a polynomial equation of least degree for each set of roots.
3. \(3, -0.5, 1\)
   \(\text{Equation: } (x - 3)(x + 0.5)(x - 1) = 0\)

4. \(3, 3, 1, 1, -2\)
   \(\text{Equation: } (x - 3)^2(x - 1)^2(x + 2) = 0\)

5. \(\pm 2i, 3, -3\)
   \(\text{Equation: } (x + 2i)(x - 2i)(x - 3)(x + 3) = 0\)

6. \(-1, 3 \pm i, 2 \pm 3i\)
   \(\text{Equation: } (x + 1)(x - 3 + i)(x - 3 - i)(x - 2 - 3i)(x - 2 + 3i) = 0\)

State the number of complex roots of each equation. Then find the roots and graph the related function.
7. \(3x - 5 = 0\)
   \(x = \frac{5}{3}\)

8. \(x^2 + 4 = 0\)
   \(x = \pm 2i\)

9. \(c^2 + 2c + 1 = 0\)
   \(c = -1\)

10. \(x^3 + 2x^2 - 15x = 0\)
    \(x = -3, 5, 0\)

11. \textbf{Real Estate} A developer wants to build homes on a rectangular plot of land 3 kilometers long and 4 kilometers wide. In this part of the city, regulations require a greenbelt of uniform width along two adjacent sides. The greenbelt must be 10 times the area of the development. Find the width of the greenbelt.
Practice

Quadratic Equations

Solve each equation by completing the square.

1. \( x^2 - 5x - \frac{11}{4} = 0 \)
2. \(-4x^2 - 11x = 7\)

Find the discriminant of each equation and describe the nature of the roots of the equation. Then solve the equation by using the Quadratic Formula.

3. \( x^2 + x - 6 = 0 \)
4. \( 4x^2 - 4x - 15 = 0 \)

5. \( 9x^2 - 12x + 4 = 0 \)
6. \( 3x^2 + 2x + 5 = 0 \)

Solve each equation.

7. \( 2x^2 + 5x - 12 = 0 \)
8. \( 5x^2 - 14x + 11 = 0 \)

9. Architecture  The ancient Greek mathematicians thought that the most pleasing geometric forms, such as the ratio of the height to the width of a doorway, were created using the golden section. However, they were surprised to learn that the golden section is not a rational number. One way of expressing the golden section is by using a line segment. In the line segment shown, \( \frac{AB}{AC} = \frac{AC}{CB} \). If \( AC = 1 \) unit, find the ratio \( \frac{AB}{AC} \).
The Remainder and Factor Theorems

Divide using synthetic division.

1. \((3x^2 + 4x - 12) \div (x + 5)\)  
2. \((x^2 - 5x - 12) \div (x - 3)\)

3. \((x^4 - 3x^2 + 12) \div (x + 1)\)  
4. \((2x^3 + 3x^2 - 8x + 3) \div (x + 3)\)

Use the Remainder Theorem to find the remainder for each division. State whether the binomial is a factor of the polynomial.

5. \((2x^4 + 4x^3 - x^2 + 9) \div (x + 1)\)  
6. \((2x^3 - 3x^2 - 10x + 3) \div (x - 3)\)

7. \((3t^3 - 10t^2 + t - 5) \div (t - 4)\)  
8. \((10x^3 - 11x^2 - 47x + 30) \div (x + 2)\)

9. \((x^4 + 5x^3 - 14x^2) \div (x - 2)\)  
10. \((2x^4 + 14x^3 - 2x^2 - 14x) \div (x + 7)\)

11. \((y^3 + y^2 - 10) \div (y + 3)\)  
12. \((n^4 - n^3 - 10n^2 + 4n + 24) \div (n + 2)\)

13. Use synthetic division to find all the factors of \(x^3 + 6x^2 - 9x - 54\) if one of the factors is \(x - 3\).

14. Manufacturing A cylindrical chemical storage tank must have a height 4 meters greater than the radius of the top of the tank. Determine the radius of the top and the height of the tank if the tank must have a volume of 15.71 cubic meters.
Practice

The Rational Root Theorem

List the possible rational roots of each equation. Then determine the rational roots.

1. \(x^3 - x^2 - 8x + 12 = 0\)

2. \(2x^3 - 3x^2 - 2x + 3 = 0\)

3. \(36x^4 - 13x^2 + 1 = 0\)

4. \(x^3 + 3x^2 - 6x - 8 = 0\)

5. \(x^4 - 3x^3 - 11x^2 + 3x + 10 = 0\)

6. \(x^4 + x^2 - 2 = 0\)

7. \(3x^3 + x^2 - 8x + 6 = 0\)

8. \(x^3 + 4x^2 - 2x + 15 = 0\)

Find the number of possible positive real zeros and the number of possible negative real zeros. Then determine the rational zeros.

9. \(f(x) = x^3 - 2x^2 - 19x + 20\)

10. \(f(x) = x^4 + x^3 - 7x^2 - x + 6\)

11. **Driving** An automobile moving at 12 meters per second on level ground begins to slow down with an acceleration of \(-1.6\) meters per second squared. The formula for the distance an object has traveled is \(d(t) = v_0t + \frac{1}{2}at^2\), where \(v_0\) is the initial velocity and \(a\) is the acceleration. For what value(s) of \(t\) does \(d(t) = 40\) meters?
Locating Zeros of a Polynomial Function

Determine between which consecutive integers the real zeros of each function are located.

1. $f(x) = 3x^3 - 10x^2 + 22x - 4$
2. $f(x) = 2x^3 + 5x^2 - 7x - 3$

3. $f(x) = 2x^3 - 13x^2 + 14x - 4$
4. $f(x) = x^3 - 12x^2 + 17x - 9$

5. $f(x) = 4x^4 - 16x^3 - 25x^2 + 196x - 146$

6. $f(x) = x^3 - 9$

Approximate the real zeros of each function to the nearest tenth.

7. $f(x) = 3x^4 + 4x^2 - 1$
8. $f(x) = 3x^3 - x + 2$

9. $f(x) = 4x^4 - 6x^2 + 1$
10. $f(x) = 2x^3 + x^2 - 1$

11. $f(x) = x^3 - 2x^2 - 2x + 3$
12. $f(x) = x^3 - 5x^2 + 4$

Use the Upper Bound Theorem to find an integral upper bound and the Lower Bound Theorem to find an integral lower bound of the zeros of each function.

13. $f(x) = 3x^4 - x^3 - 8x^2 - 3x - 20$
14. $f(x) = 2x^3 - x^2 + x - 6$

15. For $f(x) = x^3 - 3x^2$, determine the number and type of possible complex zeros. Use the Location Principle to determine the zeros to the nearest tenth. The graph has a relative maximum at $(0, 0)$ and a relative minimum at $(2, -4)$. Sketch the graph.
Practice

Rational Equations and Partial Fractions

Solve each equation.
1. \( \frac{15}{m} - m + 8 = 10 \)
2. \( \frac{4}{b - 3} + \frac{3}{b} = \frac{-2b}{b - 3} \)
3. \( \frac{1}{2n} + \frac{6n - 9}{3n} = \frac{2}{n} \)
4. \( t - \frac{4}{t} = 3 \)
5. \( \frac{3a}{2a + 1} - \frac{4}{2a - 1} = 1 \)
6. \( \frac{2p}{p + 1} + \frac{3}{p - 1} = \frac{15 - p}{p^2 - 1} \)

Decompose each expression into partial fractions.
7. \( \frac{-3x - 29}{x^2 - 4x - 21} \)
8. \( \frac{11x - 7}{2x^2 - 3x - 2} \)

Solve each inequality.
9. \( \frac{6}{t} + 3 > \frac{2}{t} \)
10. \( \frac{2n + 1}{3n + 1} \leq \frac{n - 1}{3n + 1} \)
11. \( 1 + \frac{3y}{1 - y} > 2 \)
12. \( \frac{2x}{4} - \frac{5x + 1}{3} > 3 \)

13. **Commuting**  
Rosea drives her car 30 kilometers to the train station, where she boards a train to complete her trip. The total trip is 120 kilometers. The average speed of the train is 20 kilometers per hour faster than that of the car. At what speed must she drive her car if the total time for the trip is less than 2.5 hours?
Practice

Radical Equations and Inequalities

Solve each equation.
1. \( \sqrt{x} - 2 = 6 \)
2. \( \sqrt[3]{x^2} - 1 = 3 \)

3. \( \sqrt[7]{7r} + 5 = -3 \)
4. \( \sqrt[6]{x + 12} - \sqrt[4]{x + 9} = 1 \)

5. \( \sqrt{x} - 3 - 3\sqrt{x + 12} = -11 \)
6. \( \sqrt[6]{n} - 3 = \sqrt[4]{4 + 7n} \)

7. \( 5 + 2x = \sqrt{x^2 - 2x + 1} \)
8. \( 3 - \sqrt{r + 1} = \sqrt[4]{4 - r} \)

Solve each inequality.
9. \( \sqrt[3]{3r} + 5 > 1 \)
10. \( \sqrt[2]{2t} - 3 < 5 \)

11. \( \sqrt{2m} + 3 > 5 \)
12. \( \sqrt[3]{3x + 5} < 9 \)

13. **Engineering**  A team of engineers must design a fuel tank in the shape of a cone. The surface area of a cone (excluding the base) is given by the formula \( S = \pi \sqrt{r^2 + h^2} \). Find the radius of a cone with a height of 21 meters and a surface area of 155 meters squared.
Write a polynomial function to model each set of data.

1. The farther a planet is from the Sun, the longer it takes to complete an orbit.
   
   | Distance (AU) | 0.39 | 0.72 | 1.00 | 1.49 | 5.19 | 9.51 | 19.1 | 30.0 | 39.3 |
   | Period (days)  | 88   | 225  | 365  | 687  | 4344 | 10,775 | 30,681 | 60,267 | 90,582 |


2. The amount of food energy produced by farms increases as more energy is expended. The following table shows the amount of energy produced and the amount of energy expended to produce the food.

   | Energy Input (Calories) | 606 | 970 | 1121 | 1227 | 1318 | 1455 | 1636 | 2030 | 2182 | 2242 |
   | Energy Output (Calories) | 133 | 144 | 148 | 157 | 171 | 175 | 187 | 193 | 198 | 198 |

   Source: NSTA Energy-Environment Source Book.

3. The temperature of Earth’s atmosphere varies with altitude.

   | Altitude (km) | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
   | Temperature (K) | 293 | 228 | 217 | 235 | 254 | 269 | 244 | 207 | 178 | 178 |

   Source: Living in the Environment, by Miller G. Tyler.

4. Water quality varies with the season. This table shows the average hardness (amount of dissolved minerals) of water in the Missouri River measured at Kansas City, Missouri.

   | Hardness (CaCO₃ ppm) | 310 | 250 | 180 | 175 | 230 | 175 | 170 | 180 | 210 | 230 | 295 | 300 |

Practice

Angles and Degree Measure

Change each measure to degrees, minutes, and seconds.

1. 28.955°

2. −57.327°

Write each measure as a decimal degree to the nearest thousandth.

3. 32° 28′ 10″

4. −73° 14′ 35″

Give the angle measure represented by each rotation.

5. 1.5 rotations clockwise

6. 2.6 rotations counterclockwise

Identify all angles that are coterminal with each angle. Then find one positive angle and one negative angle that are coterminal with each angle.

7. 43°

8. −30°

If each angle is in standard position, determine a coterminal angle that is between 0° and 360°, and state the quadrant in which the terminal side lies.

9. 472°

10. −995°

Find the measure of the reference angle for each angle.

11. 227°

12. 640°

13. Navigation   For an upcoming trip, Jackie plans to sail from Santa Barbara Island, located at 33° 28′ 32″ N, 119° 2′ 7″ W, to Santa Catalina Island, located at 33.386° N, 118.430° W. Write the latitude and longitude for Santa Barbara Island as decimals to the nearest thousandth and the latitude and longitude for Santa Catalina Island as degrees, minutes, and seconds.
Trigonometric Ratios in Right Triangles

Find the values of the sine, cosine, and tangent for each \( \angle B \).

1. \( \triangle ABC \) with sides 8 in., 3 in., and unknown.

2. \( \triangle ABC \) with sides 16 cm, 8 cm, and unknown.

3. If \( \tan \theta = 5 \), find \( \cot \theta \).

4. If \( \sin \theta = \frac{3}{8} \), find \( \csc \theta \).

Find the values of the six trigonometric ratios for each \( \angle S \).

5. \( \triangle QRS \) with sides 21 m, 7 m, and unknown.

6. \( \triangle QRS \) with sides 19 in., 7 in., and unknown.

7. Physics  Suppose you are traveling in a car when a beam of light passes from the air to the windshield. The measure of the angle of incidence is 55\(^\circ\), and the measure of the angle of refraction is 35\(^\circ\) 15\('\). Use Snell’s Law, \( \frac{\sin \theta_i}{\sin \theta_r} = n \), to find the index of refraction \( n \) of the windshield to the nearest thousandth.
Practice

Trigonometric Functions on the Unit Circle

Use the unit circle to find each value.

1. \( \csc 90^\circ \)

2. \( \tan 270^\circ \)

3. \( \sin (-90^\circ) \)

Use the unit circle to find the values of the six trigonometric functions for each angle.

4. \( 45^\circ \)

5. \( 120^\circ \)

Find the values of the six trigonometric functions for angle \( \theta \) in standard position if a point with the given coordinates lies on its terminal side.

6. \((-1, 5)\)

7. \((7, 0)\)

8. \((-3, -4)\)
Practice

Applying Trigonometric Functions

Solve each problem. Round to the nearest tenth.

1. If \( A = 55^\circ 55' \) and \( c = 16 \), find \( a \).

2. If \( a = 9 \) and \( B = 49^\circ \), find \( b \).

3. If \( B = 56^\circ 48' \) and \( c = 63.1 \), find \( b \).

4. If \( B = 64^\circ \) and \( b = 19.2 \), find \( a \).

5. If \( b = 14 \) and \( A = 16^\circ \), find \( c \).

6. **Construction** A 30-foot ladder leaning against the side of a house makes a \( 70^\circ 5' \) angle with the ground.

   a. How far up the side of the house does the ladder reach?

   b. What is the horizontal distance between the bottom of the ladder and the house?

7. **Geometry** A circle is circumscribed about a regular hexagon with an apothem of 4.8 centimeters.

   a. Find the radius of the circumscribed circle.

   b. What is the length of a side of the hexagon?

   c. What is the perimeter of the hexagon?

8. **Observation** A person standing 100 feet from the bottom of a cliff notices a tower on top of the cliff. The angle of elevation to the top of the cliff is \( 30^\circ \). The angle of elevation to the top of the tower is \( 58^\circ \). How tall is the tower?
Practice

Solving Right Triangles

Solve each equation if $0^\circ \leq x \leq 360^\circ$.

1. $\cos x = \frac{\sqrt{2}}{2}$

2. $\tan x = 1$

3. $\sin x = \frac{1}{2}$

Evaluate each expression. Assume that all angles are in Quadrant I.

4. $\tan \left( \tan^{-1} \frac{\sqrt{3}}{3} \right)$

5. $\tan \left( \cos^{-1} \frac{2}{3} \right)$

6. $\cos \left( \arcsin \frac{5}{13} \right)$

Solve each problem. Round to the nearest tenth.

7. If $q = 10$ and $s = 3$, find $S$.

8. If $r = 12$ and $s = 4$, find $R$.

9. If $q = 20$ and $r = 15$, find $S$.

Solve each triangle described, given the triangle at the right. Round to the nearest tenth, if necessary.

10. $a = 9$, $B = 49^\circ$

11. $A = 16^\circ$, $c = 14$

12. $a = 2$, $b = 7$

13. Recreation The swimming pool at Perris Hill Plunge is 50 feet long and 25 feet wide. The bottom of the pool is slanted so that the water depth is 3 feet at the shallow end and 15 feet at the deep end. What is the angle of elevation at the bottom of the pool?
Practice

The Law of Sines

Solve each triangle. Round to the nearest tenth.
1. $A = 38^\circ, B = 63^\circ, c = 15$
2. $A = 33^\circ, B = 29^\circ, b = 41$
3. $A = 150^\circ, C = 20^\circ, a = 200$
4. $A = 30^\circ, B = 45^\circ, a = 10$

Find the area of each triangle. Round to the nearest tenth.
5. $c = 4, A = 37^\circ, B = 69^\circ$
6. $C = 85^\circ, a = 2, B = 19^\circ$
7. $A = 50^\circ, b = 12, c = 14$
8. $b = 14, C = 110^\circ, B = 25^\circ$
9. $b = 15, c = 20, A = 115^\circ$
10. $a = 68, c = 110, B = 42.5^\circ$

11. **Street Lighting** A lamppost tilts toward the sun at a $2^\circ$ angle from the vertical and casts a 25-foot shadow. The angle from the tip of the shadow to the top of the lamppost is $45^\circ$. Find the length of the lamppost.
The Ambiguous Case for the Law of Sines

**Determine the number of possible solutions for each triangle.**

1. $A = 42^\circ, a = 22, b = 12$
2. $a = 15, b = 25, A = 85^\circ$
3. $A = 58^\circ, a = 4.5, b = 5$
4. $A = 110^\circ, a = 4, c = 4$

**Find all solutions for each triangle. If no solutions exist, write none. Round to the nearest tenth.**

5. $b = 50, a = 33, A = 132^\circ$
6. $a = 125, A = 25^\circ, b = 150$
7. $a = 32, c = 20, A = 112^\circ$
8. $a = 12, b = 15, A = 55^\circ$
9. $A = 42^\circ, a = 22, b = 12$
10. $b = 15, c = 13, C = 50^\circ$
11. **Property Maintenance** The McDougalls plan to fence a triangular parcel of their land. One side of the property is 75 feet in length. It forms a $38^\circ$ angle with another side of the property, which has not yet been measured. The remaining side of the property is 95 feet in length. Approximate to the nearest tenth the length of fence needed to enclose this parcel of the McDougalls’ lot.
Practice

The Law of Cosines

Solve each triangle. Round to the nearest tenth.

1. \(a = 20, b = 12, c = 28\)
2. \(a = 10, c = 8, B = 100^\circ\)

3. \(c = 49, b = 40, A = 53^\circ\)
4. \(a = 5, b = 7, c = 10\)

Find the area of each triangle. Round to the nearest tenth.

5. \(a = 5, b = 12, c = 13\)
6. \(a = 11, b = 13, c = 16\)

7. \(a = 14, b = 9, c = 8\)
8. \(a = 8, b = 7, c = 3\)

9. The sides of a triangle measure 13.4 centimeters, 18.7 centimeters, and 26.5 centimeters. Find the measure of the angle with the least measure.

10. **Orienteering** During an orienteering hike, two hikers start at point \(A\) and head in a direction 30° west of south to point \(B\). They hike 6 miles from point \(A\) to point \(B\). From point \(B\), they hike to point \(C\) and then from point \(C\) back to point \(A\), which is 8 miles directly north of point \(C\). How many miles did they hike from point \(B\) to point \(C\)?
Practice

Angles and Radian Measure

Change each degree measure to radian measure in terms of $\pi$.

1. $-250^\circ$  
2. $6^\circ$  
3. $-145^\circ$

4. $870^\circ$  
5. $18^\circ$  
6. $-820^\circ$

Change each radian measure to degree measure. Round to the nearest tenth, if necessary.

7. $4\pi$  
8. $\frac{13\pi}{30}$

9. $-1$

10. $\frac{3\pi}{16}$  
11. $-2.56$  
12. $-\frac{7\pi}{9}$

Evaluate each expression.

13. $\tan \frac{\pi}{4}$  
14. $\cos \frac{3\pi}{2}$  
15. $\sin \frac{3\pi}{2}$

16. $\tan \frac{11\pi}{6}$  
17. $\cos \frac{3\pi}{4}$  
18. $\sin \frac{5\pi}{3}$

Given the measurement of a central angle, find the length of its intercepted arc in a circle of radius 10 centimeters. Round to the nearest tenth.

19. $\frac{\pi}{6}$  
20. $\frac{3\pi}{5}$  
21. $\frac{\pi}{2}$

Find the area of each sector, given its central angle $\theta$ and the radius of the circle. Round to the nearest tenth.

22. $\theta = \frac{\pi}{6}$, $r = 14$  
23. $\theta = \frac{7\pi}{4}$, $r = 4$


**Linear and Angular Velocity**

Determine each angular displacement in radians. Round to the nearest tenth.

1. 6 revolutions
2. 4.3 revolutions
3. 85 revolutions
4. 11.5 revolutions
5. 7.7 revolutions
6. 17.8 revolutions

Determine each angular velocity. Round to the nearest tenth.

7. 2.6 revolutions in 6 seconds
8. 7.9 revolutions in 11 seconds
9. 118.3 revolutions in 19 minutes
10. 5.5 revolutions in 4 minutes
11. 22.4 revolutions in 15 seconds
12. 14 revolutions in 2 minutes

Determine the linear velocity of a point rotating at the given angular velocity at a distance \( r \) from the center of the rotating object. Round to the nearest tenth.

13. \( \omega = 14.3 \) radians per second, \( r = 7 \) centimeters
14. \( \omega = 28 \) radians per second, \( r = 2 \) feet
15. \( \omega = 5.4\pi \) radians per minute, \( r = 1.3 \) meters
16. \( \omega = 41.7\pi \) radians per second, \( r = 18 \) inches
17. \( \omega = 234 \) radians per minute, \( r = 31 \) inches

18. **Clocks** Suppose the second hand on a clock is 3 inches long. Find the linear velocity of the tip of the second hand.
Graphing Sine and Cosine Functions

Find each value by referring to the graph of the sine or the cosine function.

1. $\cos \pi$
2. $\sin \frac{3\pi}{2}$
3. $\sin \left( -\frac{7\pi}{2} \right)$

Find the values of $\theta$ for which each equation is true.

4. $\sin \theta = 0$
5. $\cos \theta = 1$
6. $\cos \theta = -1$

Graph each function for the given interval.

7. $y = \sin x; -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
8. $y = \cos x; 7\pi \leq x \leq 9\pi$

Determine whether each graph is $y = \sin x$, $y = \cos x$, or neither.

9.
10.

11. Meteorology The equation $y = 70.5 + 19.5 \sin \left( \frac{\pi}{6}(t - 4) \right)$ models the average monthly temperature for Phoenix, Arizona, in degrees Fahrenheit. In this equation, $t$ denotes the number of months, with $t = 1$ representing January. What is the average monthly temperature for July?
Amplitude and Period of Sine and Cosine Functions

State the amplitude and period for each function. Then graph each function.

1. \( y = -2 \sin \theta \)

2. \( y = 4 \cos \frac{\theta}{3} \)

3. \( y = 1.5 \cos 4\theta \)

4. \( y = -\frac{2}{3} \sin \frac{\theta}{2} \)

Write an equation of the sine function with each amplitude and period.

5. amplitude = 3, period = \( 2\pi \)

6. amplitude = 8.5, period = \( 6\pi \)

Write an equation of the cosine function with each amplitude and period.

7. amplitude = 0.5, period = \( 0.2\pi \)

8. amplitude = \( \frac{1}{5} \), period = \( \frac{2}{5}\pi \)

9. **Music** A piano tuner strikes a tuning fork for note A above middle C and sets in motion vibrations that can be modeled by the equation \( y = 0.001 \sin 880\pi t \). Find the amplitude and period for the function.
Translations of Sine and Cosine Functions

State the vertical shift and the equation of the midline for each function. Then graph each function.
1. $y = 4 \cos \theta + 4$
2. $y = \sin 2\theta - 2$

State the amplitude, period, phase shift, and vertical shift for each function. Then graph the function.
3. $y = 2 \sin \left(\theta + \frac{\pi}{2}\right) - 3$
4. $y = \frac{1}{2} \cos (2\theta - \pi) + 2$

Write an equation of the specified function with each amplitude, period, phase shift, and vertical shift.
5. Sine function: amplitude = 15, period = $4\pi$, phase shift = $\frac{\pi}{2}$, vertical shift = $-10$

6. Cosine function: amplitude = $\frac{2}{3}$, period = $\frac{\pi}{3}$, phase shift = $-\frac{\pi}{3}$, vertical shift = 5

7. Sine function: amplitude = 6, period = $\pi$, phase shift = 0, vertical shift = $-\frac{3}{2}$
1. **Meteorology**  The average monthly temperatures in degrees Fahrenheit (°F) for Baltimore, Maryland, are given below.

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>June</th>
<th>July</th>
<th>Aug</th>
<th>Sept</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>32</td>
<td>35</td>
<td>44</td>
<td>53</td>
<td>63</td>
<td>73</td>
<td>77</td>
<td>76</td>
<td>69</td>
<td>57</td>
<td>47</td>
<td>37</td>
</tr>
</tbody>
</table>

a. Find the amplitude of a sinusoidal function that models the monthly temperatures.

b. Find the vertical shift of a sinusoidal function that models the monthly temperatures.

c. What is the period of a sinusoidal function that models the monthly temperatures?

d. Write a sinusoidal function that models the monthly temperatures, using $t = 1$ to represent January.

e. According to your model, what is the average temperature in July? How does this compare with the actual average?

f. According to your model, what is the average temperature in December? How does this compare with the actual average?

2. **Boating**  A buoy, bobbing up and down in the water as waves move past it, moves from its highest point to its lowest point and back to its highest point every 10 seconds. The distance between its highest and lowest points is 3 feet.

a. What is the amplitude of a sinusoidal function that models the bobbing buoy?

b. What is the period of a sinusoidal function that models the bobbing buoy?

c. Write a sinusoidal function that models the bobbing buoy, using $t = 0$ at its highest point.

d. According to your model, what is the height of the buoy at $t = 2$ seconds?

e. According to your model, what is the height of the buoy at $t = 6$ seconds?
Graphing Other Trigonometric Functions

Find each value by referring to the graphs of the trigonometric functions.

1. \[ \tan \left( -\frac{3\pi}{2} \right) \]
2. \[ \cot \left( \frac{3\pi}{2} \right) \]

3. \[ \sec 4\pi \]
4. \[ \csc \left( -\frac{7\pi}{2} \right) \]

Find the values of \( \theta \) for which each equation is true.

5. \[ \tan \theta = 0 \]
6. \[ \cot \theta = 0 \]

7. \[ \csc \theta = 1 \]
8. \[ \sec \theta = -1 \]

Graph each function.

9. \[ y = \tan \left( 2\theta + \pi \right) + 1 \]

10. \[ y = \cot \left( \frac{\theta}{2} - \frac{\pi}{2} \right) - 2 \]

11. \[ y = \csc \theta + 3 \]

12. \[ y = \sec \left( \frac{\theta}{3} + \pi \right) - 1 \]
Write the equation for the inverse of each function. Then graph the function and its inverse.

1. \( y = \tan 2x \)  
2. \( y = \frac{\pi}{2} + \arccos x \)

Find each value.

3. \( \arccos (-1) \)  
4. \( \arctan 1 \)  
5. \( \arcsin \left( -\frac{1}{2} \right) \)

6. \( \sin^{-1} \left( \frac{\sqrt{3}}{2} \right) \)  
7. \( \cos^{-1} \left( \sin \frac{\pi}{3} \right) \)  
8. \( \tan \left( \sin^{-1} 1 - \cos^{-1} \frac{1}{2} \right) \)

9. **Weather**  The equation \( y = 10 \sin \left( \frac{\pi}{6}t - \frac{2\pi}{3} \right) + 57 \) models the average monthly temperatures for Napa, California. In this equation, \( t \) denotes the number of months with January represented by \( t = 1 \). During which two months is the average temperature 62°F?
Use the given information to determine the exact trigonometric value if $0^\circ < \theta < 90^\circ$.

1. If $\cos \theta = \frac{1}{4}$, find $\tan \theta$.
2. If $\sin \theta = \frac{2}{3}$, find $\cos \theta$.
3. If $\tan \theta = \frac{7}{2}$, find $\sin \theta$.
4. If $\tan \theta = 2$, find $\cot \theta$.

Express each value as a trigonometric function of an angle in Quadrant I.

5. $\cos 892^\circ$
6. $\csc 495^\circ$
7. $\sin \frac{23\pi}{3}$

Simplify each expression.

8. $\cos x + \sin x \tan x$
9. $\frac{\cot A}{\tan A}$
10. $\sin^2 \theta \cos^2 \theta - \cos^2 \theta$

11. **Kite Flying**
    Brett and Tara are flying a kite. When the string is tied to the ground, the height of the kite can be determined by the formula $\frac{L}{H} = \csc \theta$, where $L$ is the length of the string and $\theta$ is the angle between the string and the level ground. What formula could Brett and Tara use to find the height of the kite if they know the value of $\sin \theta$?
7-2

Practice

Verifying Trigonometric Identities

Verify that each equation is an identity.

1. \( \frac{\csc x}{\cot x + \tan x} = \cos x \)

2. \( \frac{1}{\sin y - 1} - \frac{1}{\sin y + 1} = -2 \sec^2 y \)

3. \( \sin^3 x - \cos^3 x = (1 + \sin x \cos x)(\sin x - \cos x) \)

4. \( \tan \theta + \frac{\cos \theta}{1 + \sin \theta} = \sec \theta \)

Find a numerical value of one trigonometric function of \( x \).

5. \( \sin x \cot x = 1 \)  
6. \( \sin x = 3 \cos x \)  
7. \( \cos x = \cot x \)

8. Physics  The work done in moving an object is given by the formula

\[ W = Fd \cos \theta, \]

where \( d \) is the displacement, \( F \) is the force exerted, and \( \theta \) is the angle between the displacement and the force. Verify that

\[ W = Fd \frac{\cot \theta}{\csc \theta} \]

is an equivalent formula.
Practice

Sum and Difference Identities

Use sum or difference identities to find the exact value of each trigonometric function.

1. \( \cos \frac{5\pi}{12} \)  
2. \( \sin (-165^\circ) \)  
3. \( \tan 345^\circ \) 

4. \( \csc 915^\circ \)  
5. \( \tan \left( -\frac{7\pi}{12} \right) \)  
6. \( \sec \frac{\pi}{12} \)

Find each exact value if \( 0 < x < \frac{\pi}{2} \) and \( 0 < y < \frac{\pi}{2} \).

7. \( \cos (x + y) \) if \( \sin x = \frac{5}{13} \) and \( \sin y = \frac{4}{5} \)

8. \( \sin (x - y) \) if \( \cos x = \frac{8}{17} \) and \( \cos y = \frac{3}{5} \)

9. \( \tan (x - y) \) if \( \csc x = \frac{13}{5} \) and \( \cot y = \frac{4}{3} \)

Verify that each equation is an identity.

10. \( \cos (180^\circ - \theta) = -\cos \theta \)  
11. \( \sin (360^\circ + \theta) = \sin \theta \)

12. Physics  Sound waves can be modeled by equations of the form \( y = 20 \sin (3t + \theta) \). Determine what type of interference results when sound waves modeled by the equations \( y = 20 \sin (3t + 90^\circ) \) and \( y = 20 \sin (3t + 270^\circ) \) are combined. (Hint: Refer to the application in Lesson 7-3.)
Practice

Double-Angle and Half-Angle Identities

Use a half-angle identity to find the exact value of each function.

1. \( \sin 105^\circ \)  
2. \( \tan \frac{\pi}{8} \)  
3. \( \cos \frac{5\pi}{8} \)

Use the given information to find \( \sin 2\theta \), \( \cos 2\theta \), and \( \tan 2\theta \).

4. \( \sin \theta = \frac{12}{13} \), \( 0^\circ < \theta < 90^\circ \)  
5. \( \tan \theta = \frac{1}{2} \), \( \pi < \theta < \frac{3\pi}{2} \)

6. \( \sec \theta = -\frac{5}{2} \), \( \frac{\pi}{2} < \theta < \pi \)  
7. \( \sin \theta = \frac{3}{5} \), \( 0 < \theta < \frac{\pi}{2} \)

Verify that each equation is an identity.

8. \( 1 + \sin 2x = (\sin x + \cos x)^2 \)

9. \( \cos x \sin x = \frac{\sin 2x}{2} \)

10. **Baseball** A batter hits a ball with an initial velocity \( v_0 \) of 100 feet per second at an angle \( \theta \) to the horizontal. An outfielder catches the ball 200 feet from home plate. Find \( \theta \) if the range of a projectile is given by the formula \( R = \frac{1}{32}v_0^2 \sin 2\theta \).
Solving Trigonometric Equations

Solve each equation for principal values of $x$. Express solutions in degrees.

1. $\cos x = 3 \cos x - 2$
2. $2 \sin^2 x - 1 = 0$

Solve each equation for $0^\circ \leq x < 360^\circ$.

3. $\sec^2 x + \tan x - 1 = 0$
4. $\cos 2x + 3 \cos x - 1 = 0$

Solve each equation for $0 \leq x < 2\pi$.

5. $4 \sin^2 x - 4 \sin x + 1 = 0$
6. $\cos 2x + \sin x = 1$

Solve each equation for all real values of $x$.

7. $3 \cos 2x - 5 \cos x = 1$
8. $2 \sin^2 x - 5 \sin x + 2 = 0$

9. $3 \sec^2 x - 4 = 0$
10. $\tan x (\tan x - 1) = 0$

11. Aviation An airplane takes off from the ground and reaches a height of 500 feet after flying 2 miles. Given the formula $H = d \tan \theta$, where $H$ is the height of the plane and $d$ is the distance (along the ground) the plane has flown, find the angle of ascent $\theta$ at which the plane took off.
Normal Form of a Linear Equation

Write the standard form of the equation of each line, given
p, the length of the normal segment, and ϕ, the angle the
normal segment makes with the positive x-axis.

1. \( p = 4, \phi = 30° \)
2. \( p = 2\sqrt{2}, \phi = \frac{\pi}{4} \)

3. \( p = 3, \phi = 60° \)
4. \( p = 8, \phi = \frac{5\pi}{6} \)

5. \( p = 2\sqrt{3}, \phi = \frac{7\pi}{4} \)
6. \( p = 15, \phi = 225° \)

Write each equation in normal form. Then find the length of
the normal and the angle it makes with the positive x-axis.

7. \( 3x - 2y - 1 = 0 \)

8. \( 5x + y - 12 = 0 \)

9. \( 4x + 3y - 4 = 0 \)

10. \( y = x + 5 \)

11. \( 2x + y + 1 = 0 \)

12. \( x + y - 5 = 0 \)
Practice

Distance From a Point to a Line

Find the distance between the point with the given coordinates and the line with the given equation.

1. \((-1, 5), 3x - 4y - 1 = 0\)  
2. \((2, 5), 5x - 12y + 1 = 0\)

3. \((1, -4), 12x + 5y - 3 = 0\)  
4. \((-1, -3), 6x + 8y - 3 = 0\)

Find the distance between the parallel lines with the given equations.

5. \(2x - 3y + 4 = 0\)  
\[y = \frac{2}{3}x + 5\]  
6. \(4x - y + 1 = 0\)  
\[4x - y - 8 = 0\]

Find equations of the lines that bisect the acute and obtuse angles formed by the lines with the given equations.

7. \(x + 2y - 3 = 0\)  
\[x - y + 4 = 0\]

8. \(9x + 12y + 10 = 0\)  
\[3x + 2y - 6 = 0\]
Geometric Vectors

Use a ruler and a protractor to determine the magnitude (in centimeters) and direction of each vector.

1. \( \vec{x} \)

2. \( \vec{y} \)

3. \( \vec{z} \)

Find the magnitude and direction of each resultant.

4. \( \vec{x} + \vec{y} \)

5. \( \vec{x} - \vec{z} \)

6. \( 2\vec{x} + \vec{y} \)

7. \( \vec{y} + 3\vec{z} \)

Find the magnitude of the horizontal and vertical components of each vector shown in Exercises 1-3.

8. \( \vec{x} \)

9. \( \vec{y} \)

10. \( \vec{z} \)

11. Aviation An airplane is flying at a velocity of 500 miles per hour due north when it encounters a wind blowing out of the west at 50 miles per hour. What is the magnitude of the airplane’s resultant velocity?
Algebraic Vectors

Write the ordered pair that represents \( \overrightarrow{AB} \). Then find the magnitude of \( \overrightarrow{AB} \).

1. \((2, 4), B(-1, 3)\)
2. \((4, -2), B(5, -5)\)
3. \((-3, -6), B(8, -1)\)

Find an ordered pair to represent \( \mathbf{u} \) in each equation if \( \mathbf{v} = \langle 2, -1 \rangle \) and \( \mathbf{w} = \langle -3, 5 \rangle \).

4. \( \mathbf{u} = 3\mathbf{v} \)
5. \( \mathbf{u} = \mathbf{w} - 2\mathbf{v} \)

6. \( \mathbf{u} = 4\mathbf{v} + 3\mathbf{w} \)
7. \( \mathbf{u} = 5\mathbf{w} - 3\mathbf{v} \)

Find the magnitude of each vector, and write each vector as the sum of unit vectors.

8. \( \langle 2, 6 \rangle \)
9. \( \langle 4, -5 \rangle \)

10. **Gardening**  Nancy and Harry are lifting a stone statue and moving it to a new location in their garden. Nancy is pushing the statue with a force of 120 newtons (N) at a 60° angle with the horizontal while Harry is pulling the statue with a force of 180 newtons at a 40° angle with the horizontal. What is the magnitude of the combined force they exert on the statue?
Locate point B in space. Then find the magnitude of a vector from the origin to B.
1. B(4, 7, 6)  2. B(4, −2, 6)

Write the ordered triple that represents AB. Then find the magnitude of AB.
3. A(2, 1, 3), B(−4, 5, 7)  4. A(4, 0, 6), B(7, 1, −3)
5. A(−4, 5, 8), B(7, 2, −9)  6. A(6, 8, −5), B(7, −3, 12)

Find an ordered triple to represent u in each equation if v = <2, −4, 5> and w = <6, −8, 9>.
7. u = v + w  8. u = v − w
9. u = 4v + 3w  10. u = 5v − 2w

11. Physics  Suppose that the force acting on an object can be expressed by the vector <85, 35, 110>, where each measure in the ordered triple represents the force in pounds. What is the magnitude of this force?
Practice

Perpendicular Vectors

Find each inner product and state whether the vectors are perpendicular. Write yes or no.

1. \(\langle 3, 6 \rangle \cdot \langle -4, 2 \rangle\)  
2. \(\langle -1, 4 \rangle \cdot \langle 3, -2 \rangle\)  
3. \(\langle 2, 0 \rangle \cdot \langle -1, -1 \rangle\)

4. \(\langle -2, 0, 1 \rangle \cdot \langle 3, 2, -3 \rangle\)  
5. \(\langle -4, -1, 1 \rangle \cdot \langle 1, -3, 4 \rangle\)  
6. \(\langle 0, 0, 1 \rangle \cdot \langle 1, -2, 0 \rangle\)

Find each cross product. Then verify that the resulting vector is perpendicular to the given vectors.

7. \(\langle 1, 3, 4 \rangle \times \langle -1, 0, -1 \rangle\)  
8. \(\langle 3, 1, -6 \rangle \times \langle -2, 4, 3 \rangle\)

9. \(\langle 3, 1, 2 \rangle \times \langle 2, -3, 1 \rangle\)  
10. \(\langle 4, -1, 0 \rangle \times \langle 5, -3, -1 \rangle\)

11. \(\langle -6, 1, 3 \rangle \times \langle -2, -2, 1 \rangle\)  
12. \(\langle 0, 0, 6 \rangle \times \langle 3, -2, -4 \rangle\)

13. Physics  
Janna is using a force of 100 pounds to push a cart up a ramp. The ramp is 6 feet long and is at a 30° angle with the horizontal. How much work is Janna doing in the vertical direction? (Hint: Use the sine ratio and the formula \(W = \mathbf{F} \cdot \mathbf{d}\).)
Make a sketch to show the given vectors.

1. a force of 97 newtons acting on an object while a force of 38 newtons acts on the same object at an angle of $70^\circ$ with the first force

2. a force of 85 pounds due north and a force of 100 pounds due west acting on the same object

Find the magnitude and direction of the resultant vector for each diagram.

3.

4.

5. What would be the force required to push a 200-pound object up a ramp inclined at $30^\circ$ with the ground?

6. Nadia is pulling a tarp along level ground with a force of 25 pounds directed along the tarp. If the tarp makes an angle of $50^\circ$ with the ground, find the horizontal and vertical components of the force.

7. Aviation A pilot flies a plane east for 200 kilometers, then $60^\circ$ south of east for 80 kilometers. Find the plane’s distance and direction from the starting point.
Vectors and Parametric Equations

Write a vector equation of the line that passes through point $P$ and is parallel to $\vec{a}$. Then write parametric equations of the line.

1. $P(-2, 1), \vec{a} = \langle 3, -4 \rangle$
2. $P(3, 7), \vec{a} = \langle 4, 5 \rangle$

3. $P(2, -4), \vec{a} = \langle 1, 3 \rangle$
4. $P(5, -8), \vec{a} = \langle 9, 2 \rangle$

Write parametric equations of the line with the given equation.

5. $y = 3x - 8$
6. $y = -x + 4$

7. $3x - 2y = 6$
8. $5x + 4y = 20$

Write an equation in slope-intercept form of the line with the given parametric equations.

9. $x = 2t + 3$
   $y = t - 4$

10. $x = t + 5$
    $y = -3t$

11. Physical Education  Brett and Chad are playing touch football in gym class. Brett has to tag Chad before he reaches a 20-yard marker. Chad follows a path defined by $\langle x - 1, y - 19 \rangle = t\langle 0, 1 \rangle$, and Brett follows a path defined by $\langle x - 12, y - 0 \rangle = t\langle -11, 19 \rangle$. Write parametric equations for the paths of Brett and Chad. Will Brett tag Chad before he reaches the 20-yard marker?
Find the initial horizontal and vertical velocity for each situation.

1. a soccer ball kicked with an initial velocity of 39 feet per second at an angle of 44° with the ground

2. a toy rocket launched from level ground with an initial velocity of 63 feet per second at an angle of 84° with the horizontal

3. a football thrown at a velocity of 10 yards per second at an angle of 58° with the ground

4. a golf ball hit with an initial velocity of 102 feet per second at an angle of 67° with the horizontal

5. Model Rocketry  Manuel launches a toy rocket from ground level with an initial velocity of 80 feet per second at an angle of 80° with the horizontal.
   a. Write parametric equations to represent the path of the rocket.
   b. How long will it take the rocket to travel 10 feet horizontally from its starting point? What will be its vertical distance at that point?

6. Sports  Jessica throws a javelin from a height of 5 feet with an initial velocity of 65 feet per second at an angle of 45° with the ground.
   a. Write parametric equations to represent the path of the javelin.
   b. After 0.5 seconds, how far has the javelin traveled horizontally and vertically?
Practice

Transformation Matrices in Three-Dimensional Space

Write the matrix for each figure.

1. 

2. 

Translate the figure in Question 1 using the given vectors. Graph each image and write the translated matrix.

3. \( \mathbf{a} \langle 1, 2, 0 \rangle \)

4. \( \mathbf{b} \langle -1, 2, -2 \rangle \)

Transform the figure in Question 2 using each matrix. Graph each image and describe the result.

5. \[
\begin{bmatrix}
2 & 0 & 0 \\
0 & 2 & 0 \\
0 & 0 & 2
\end{bmatrix}
\]

6. \[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1
\end{bmatrix}
\]
Graph each point.
1. $(2.5, 0^\circ)$
2. $(3, -135^\circ)$
3. $(-1, -30^\circ)$
4. $(-2, \frac{\pi}{4})$
5. $(1, \frac{5\pi}{4})$
6. $(2, -\frac{2\pi}{3})$

Graph each polar equation.
7. $r = 3$
8. $\theta = 60^\circ$
9. $r = 4$

Find the distance between the points with the given polar coordinates.
10. $P_1(6, 90^\circ)$ and $P_2(2, 130^\circ)$
11. $P_1(-4, 85^\circ)$ and $P_2(1, 105^\circ)$
Graph each polar equation. Identify the type of curve each represents.

1. \( r = 1 + \cos \theta \)
2. \( r = 3 \sin 3\theta \)
3. \( r = 1 + 2 \cos \theta \)
4. \( r = 2 + 2 \sin \theta \)
5. \( r = 0.5\theta \)
6. \( r^2 = 16 \cos 2\theta \)

Graph each system of polar equations. Solve the system using algebra and trigonometry. Assume \( 0 \leq \theta < 2\pi \).

7. \( r = 1 + 2 \sin \theta \)
   \( r = 2 + \sin \theta \)
8. \( r = 1 + \cos \theta \)
   \( r = 3 \cos \theta \)

9. **Design**  
   Mikaela is designing a border for her stationery. 
   Suppose she uses a rose curve. Determine an equation for designing a rose that has 8 petals with each petal 4 units long.
Practice

Polar and Rectangular Coordinates

Find the rectangular coordinates of each point with the given polar coordinates.
1. (6, 120°)  
2. (−4, 45°)  
3. \(\left(4, \frac{\pi}{6}\right)\)  
4. \(\left(0, \frac{13\pi}{3}\right)\)

Find the polar coordinates of each point with the given rectangular coordinates. Use \(0 \leq \theta < 2\pi\) and \(r \geq 0\).
5. (2, 2)  
6. (2, −3)  
7. (−3, \(\sqrt{3}\))  
8. (−5, −8)

Write each polar equation in rectangular form.
9. \(r = 4\)  
10. \(r \cos \theta = 5\)

Write each rectangular equation in polar form.
11. \(x^2 + y^2 = 9\)  
12. \(y = 3\)

13. Surveying A surveyor records the polar coordinates of the location of a landmark as \((40, 62°)\). What are the rectangular coordinates?
Polar Form of a Linear Equation

Write each equation in polar form. Round \( \phi \) to the nearest degree.

1. \( 3x + 2y = 16 \)
2. \( 3x + 4y = 15 \)
3. \( 3x - 4y = 12 \)
4. \( y = 2x - 1 \)

Write each equation in rectangular form.

5. \( 4 = r \cos \left( \theta + \frac{5\pi}{6} \right) \)
6. \( 2 = r \cos (\theta - 90^\circ) \)
7. \( 1 = r \cos \left( \theta - \frac{\pi}{4} \right) \)
8. \( 3 = r \cos (\theta + 240^\circ) \)

Graph each polar equation.

9. \( 3 = r \cos (\theta - 60^\circ) \)
10. \( 1 = r \cos \left( \theta + \frac{\pi}{3} \right) \)

11. **Landscaping**  A landscaper is designing a garden with hedges through which a straight path will lead from the exterior of the garden to the interior. If the polar coordinates of the endpoints of the path are \((20, 90^\circ)\) and \((10, 150^\circ)\), where \(r\) is measured in feet, what is the equation for the path?
Practice
Simplifying Complex Numbers

Simplify.
1. \(i^{38}\)  

2. \(i^{-17}\)

3. \((3 + 2i) + (4 + 5i)\)

4. \((-6 - 2i) - (-8 - 3i)\)

5. \((8 - i) - (4 - i)\)

6. \((1 + i)(3 - 2i)\)

7. \((2 - 3i)(5 + i)\)

8. \((4 + 5i)(4 - 5i)\)

9. \((3 + 4i)^2\)

10. \((4 + 3i) ÷ (1 - 2i)\)

11. \((2 + i) ÷ (2 - i)\)

12. \(\frac{8 - 7i}{1 - 2i}\)

13. **Physics** A fence post wrapped in two wires has two forces acting on it. Once force exerts 5.3 newtons due north and 4.1 newtons due east. The second force exerts 6.2 newtons due north and 2.8 newtons due east. Find the resultant force on the fence post. Write your answer as a complex number. (Hint: A vector with a horizontal component of magnitude \(a\) and a vertical component of magnitude \(b\) can be represented by the complex number \(a + bi\).)
The Complex Plane and Polar Form of Complex Numbers

Graph each number in the complex plan and find its absolute value.

1. \( z = 3i \)
2. \( z = 5 + i \)
3. \( z = -4 - 4i \)

Express each complex number in polar form.

4. \( 3 + 4i \)
5. \( -4 + 3i \)
6. \( -1 + i \)
7. \( 1 - i \)

Graph each complex number. Then express it in rectangular form.

8. \( 2 \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) \)
9. \( 4 \left( \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right) \)
10. \( 3 \left( \cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right) \)

11. **Vectors**  The force on an object is represented by the complex number \( 8 + 21i \), where the components are measured in pounds. Find the magnitude and direction of the force.
Practice

Products and Quotients of Complex Numbers in Polar Form

Find each product or quotient. Express the result in rectangular form.

1. \(3(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}) \cdot 3(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3})\)

2. \(6(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}) \div 2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})\)

3. \(14(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}) \div 2(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})\)

4. \(3(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}) \cdot 6(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})\)

5. \(2(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}) \cdot 2(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3})\)

6. \(15(\cos \pi + i \sin \pi) \div 3(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})\)

7. Electricity  Find the current in a circuit with a voltage of 12 volts and an impedance of \(2 - 4j\) ohms. Use the formula, \(E = I \cdot Z\), where \(E\) is the voltage measured in volts, \(I\) is the current measured in amperes, and \(Z\) is the impedance measured in ohms.
   (Hint: Electrical engineers use \(j\) as the imaginary unit, so they write complex numbers in the form \(a + bj\). Express each number in polar form, substitute values into the formula, and then express the current in rectangular form.)
Practice

Powers and Roots of Complex Numbers

Find each power. Express the result in rectangular form.

1. \((-2 - 2\sqrt{3}i)^3\) 
2. \((1 - i)^5\)

3. \((-1 + \sqrt{3}i)^{12}\) 
4. \(\left[1\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)\right]^{-3}\)

5. \((2 + 3i)^6\) 
6. \((1 + i)^8\)

Find each principal root. Express the result in the form \(a + bi\) with \(a\) and \(b\) rounded to the nearest hundredth.

7. \((-27i)^{\frac{1}{3}}\) 
8. \((8 - 8i)^{\frac{1}{3}}\)

9. \(\sqrt{-243i}\) 
10. \((-i)^{\frac{1}{3}}\)

11. \(\sqrt[3]{-8i}\) 
12. \(\sqrt[3]{-2 - 2\sqrt{3}i}\)
Practice

Introduction to Analytic Geometry

Find the distance between each pair of points with the given coordinates. Then find the midpoint of the segment that has endpoints at the given coordinates.

1. \((-2, 1), (3, 4)\)  
2. \((1, 1), (9, 7)\)

3. \((3, -4), (5, 2)\)  
4. \((-1, 2), (5, 4)\)

5. \((-7, -4), (2, 8)\)  
6. \((-4, 10), (4, -5)\)

Determine whether the quadrilateral having vertices with the given coordinates is a parallelogram.

7. \((4, 4), (2, -2), (-5, -1), (-3, 5)\)  
8. \((3, 5), (-1, 1), (-6, 2), (-3, 7)\)

9. \((4, -1), (2, -5), (-3, -3), (-1, 1)\)  
10. \((2, 6), (1, 2), (-4, 4), (-3, 9)\)

11. **Hiking** Jenna and Maria are hiking to a campsite located at \((2, 1)\) on a map grid, where each side of a square represents 2.5 miles. If they start their hike at \((-3, 1)\), how far must they hike to reach the campsite?
Circles

Write the standard form of the equation of each circle described. Then graph the equation.

1. center at (3, 3) tangent to the x-axis

2. center at (2, −1), radius 4

Write the standard form of each equation. Then graph the equation.

3. \( x^2 + y^2 - 8x - 6y + 21 = 0 \)

4. \( 4x^2 + 4y^2 + 16x - 8y - 5 = 0 \)

Write the standard form of the equation of the circle that passes through the points with the given coordinates. Then identify the center and radius.

5. \((-3, -2), (-2, -3), (-4, -3)\)

6. \((0, -1), (2, -3), (4, -1)\)

7. Geometry A square inscribed in a circle and centered at the origin has points at \((2, 2), (-2, 2), (2, -2)\) and \((-2, -2)\). What is the equation of the circle that circumscribes the square?
Ellipses

Write the equation of each ellipse in standard form. Then find the coordinates of its foci.

1.

2.

For the equation of each ellipse, find the coordinates of the center, foci, and vertices. Then graph the equation.

3. $4x^2 + 9y^2 - 8x - 36y + 4 = 0$

4. $25x^2 + 9y^2 - 50x - 90y + 25 = 0$

Write the equation of the ellipse that meets each set of conditions.

5. The center is at $(1, 3)$, the major axis is parallel to the $y$-axis, and one vertex is at $(1, 8)$, and $b = 3$.

6. The foci are at $(-2, 1)$ and $(-2, -7)$, and $a = 5$.

7. Construction  A semi elliptical arch is used to design a headboard for a bed frame. The headboard will have a height of 2 feet at the center and a width of 5 feet at the base. Where should the craftsman place the foci in order to sketch the arch?
Hyperbolas

For each equation, find the coordinates of the center, foci, and vertices, and the equations of the asymptotes of its graph. Then graph the equation.

1. \( x^2 - 4y^2 - 4x + 24y - 36 = 0 \)

2. \( y^2 - 4x^2 + 8x = 20 \)

Write the equation of each hyperbola.

3. 

4. 

5. Write an equation of the hyperbola for which the length of the transverse axis is 8 units, and the foci are at (6, 0) and (-4, 0).

6. Environmental Noise  Two neighbors who live one mile apart hear an explosion while they are talking on the telephone. One neighbor hears the explosion two seconds before the other. If sound travels at 1100 feet per second, determine the equation of the hyperbola on which the explosion was located.
Parabolas

For the equation of each parabola, find the coordinates of the vertex and focus, and the equations of the directrix and axis of symmetry. Then graph the equation.

1. \(x^2 - 2x - 8y + 17 = 0\)

2. \(y^2 + 6y + 9 = 12 - 12x\)

Write the equation of the parabola that meets each set of conditions. Then graph the equation.

3. The vertex is at \((-2, 4)\) and the focus is at \((-2, 3)\).

4. The focus is at \((2, 1)\), and the equation of the directrix is \(x = -2\).

5. **Satellite Dish** Suppose the receiver in a parabolic dish antenna is 2 feet from the vertex and is located at the focus. Assume that the vertex is at the origin and that the dish is pointed upward. Find an equation that models a cross section of the dish.
Rectangular and Parametric Forms of Conic Sections

Identify the conic section represented by each equation. Then write the equation in standard form and graph the equation.

1. \(x^2 - 4y + 4 = 0\)
2. \(x^2 + y^2 - 6x - 6y - 18 = 0\)

3. \(4x^2 - y^2 - 8x + 6y = 9\)
4. \(9x^2 + 5y^2 + 18x = 36\)

Find the rectangular equation of the curve whose parametric equations are given. Then graph the equation using arrows to indicate orientation.

5. \(x = 3 \cos t, y = 3 \sin t, 0 \leq t \leq 2\pi\)
6. \(x = -4 \cos t, y = 5 \sin t, 0 \leq t \leq 2\pi\)
Practice

Transformations of Conics

Identify the graph of each equation. Write an equation of the translated or rotated graph in general form.

1. \(2x^2 + 5y^2 = 9\) for \(T_{(-2, 1)}\) 
2. \(2x^2 - 4x + 3 - y = 0\) for \(T_{(1, -1)}\)

3. \(xy = 1, \theta = \frac{\pi}{4}\) 
4. \(x^2 - 4y = 0, \theta = 90^\circ\)

Identify the graph of each equation. Then find \(\theta\) to the nearest degree.

5. \(2x^2 + 2y^2 - 2x = 0\) 
6. \(3x^2 + 8xy + 4y^2 - 7 = 0\)

7. \(16x^2 - 24xy + 9y^2 - 30x - 40y = 0\) 
8. \(13x^2 - 8xy + 7y^2 - 45 = 0\)

9. **Communications** Suppose the orientation of a satellite dish that monitors radio waves is modeled by the equation \(4x^2 + 2xy + 4y^2 + \sqrt{2}x - \sqrt{2}y = 0\). What is the angle of rotation of the satellite dish about the origin?
Systems of Second-Degree Equations and Inequalities

Solve each system of equations algebraically. Round to the nearest tenth. Check the solutions by graphing each system.

1. \(2x - y = 8\)
   \(x^2 + y^2 = 9\)

2. \(x^2 - y^2 = 4\)
   \(y = 1\)

3. \(xy = 4\)
   \(x^2 = y^2 + 1\)

4. \(x^2 + y^2 = 4\)
   \(4x^2 + 9y^2 = 36\)

Graph each system of inequalities.

5. \(3 \geq (y - 1)^2 + 2x\)
   \(y \geq -3x + 1\)

6. \((x - 1)^2 + (y - 2)^2 < 9\)
   \(4(y + 1)^2 + x^2 \leq 16\)

7. Sales  Vincent’s Pizzeria reduced prices for large specialty pizzas by $5 for 1 week in March. In the previous week, sales for large specialty pizzas totaled $400. During the sale week, the number of large pizzas sold increased by 20 and total sales amounted to $600. Write a system of second-degree equations to model this situation. Find the regular price and the sale price of large specialty pizzas.
Practice

Rational Exponents

Evaluate each expression.

1. \( \frac{8^3}{8^{1}} \)
2. \( \left( \frac{4}{5} \right)^{-2} \)
3. \( 343^{\frac{2}{3}} \)

4. \( \sqrt[3]{8} \)
5. \( \sqrt{5} \cdot \sqrt{10} \)
6. \( 9^{\frac{1}{2}} \)

Simplify each expression.

7. \( (5n^3)^2 \cdot n^{-6} \)
8. \( \left( \frac{x^2}{4y^{-2}} \right)^{\frac{1}{2}} \)
9. \( (64x^6)^{\frac{1}{3}} \)

10. \( (5x^6y^4)^{\frac{1}{2}} \)
11. \( \sqrt{x^2y^3} \cdot \sqrt{x^3y^4} \)
12. \( \left( \frac{p^{6a}}{p^{-3a}} \right)^{\frac{1}{3}} \)

Express each using rational exponents.

13. \( \sqrt{x^5y^6} \)
14. \( \sqrt[5]{27x^{10}y^5} \)
15. \( \sqrt[10]{144x^6y^{10}} \)

16. \( 21\sqrt[3]{c^7} \)
17. \( \sqrt[4]{1024a^3} \)
18. \( \sqrt[5]{36a^8b^5} \)

Express each using radicals.

19. \( 64^{\frac{1}{3}} \)
20. \( 2^{\frac{1}{2}}a^{\frac{3}{2}}b^{\frac{5}{2}} \)
21. \( s^{\frac{2}{3}}t^{\frac{1}{2}}v^{\frac{2}{3}} \)

22. \( y^{\frac{3}{2}} \)
23. \( x^8y^{\frac{3}{5}} \)
24. \( (x^6y^3)^{\frac{1}{2}}z^{\frac{3}{2}} \)
Exponential Functions

**Graph each exponential function or inequality.**

1. \( y = 2^{x-1} \)

2. \( y = 4^{-x+2} \)

3. \( y > -3^x + 1 \)

4. \( y \geq 0.5^x \)

5. **Demographics**  An area in North Carolina known as The Triangle is principally composed of the cities of Durham, Raleigh, and Chapel Hill. The Triangle had a population of 700,000 in 1990. The average yearly rate of growth is 5.9%. Find the projected population for 2010.

6. **Finance**  Determine the amount of money in a savings account that provides an annual rate of 4% compounded monthly if the initial investment is $1000 and the money is left in the account for 5 years.

7. **Investments**  How much money must be invested by Mr. Kaufman if he wants to have $20,000 in his account after 15 years? He can earn 5% compounded quarterly.
1. **Demographics**  In 1995, the population of Kalamazoo, Michigan, was 79,089. This figure represented a 0.4% annual decline from 1990.
   a. Let $t$ be the number of years since 1995 and write a function that models the population in Kalamazoo in 1995.
   
   b. Predict the population in 2010 and 2015. Assume a steady rate of decline.

2. **Biology**  Suppose a certain type of bacteria reproduces according to the model $P(t) = 100e^{0.271t}$, where $t$ is time in hours.
   a. At what rate does this type of bacteria reproduce?
   
   b. What was the initial number of bacteria?
   
   c. Find the number of bacteria at $P(5)$, $P(10)$, $P(24)$, and $P(72)$. Round to the nearest whole number.

3. **Finance**  Suppose Karyn deposits $1500 in a savings account that earns 6.75% interest compounded continuously. She plans to withdraw the money in 6 years to make a $2500 down payment on a car. Will there be enough funds in Karyn’s account in 6 years to meet her goal?

4. **Banking**  Given the original principal, the annual interest rate, the amount of time for each investment, and the type of compounded interest, find the amount at the end of the investment.
   a. $P = 1250$, $r = 8.5\%$, $t = 3$ years, semiannually
   
   b. $P = 2575$, $r = 6.25\%$, $t = 5$ years 3 months, continuously
Logarithmic Functions

Write each equation in exponential form.
1. \( \log_3 81 = 4 \)
2. \( \log_8 2 = \frac{1}{3} \)
3. \( \log_{10} \frac{1}{100} = -2 \)

Write each equation in logarithmic form.
4. \( 3^3 = 27 \)
5. \( 5^{-3} = \frac{1}{125} \)
6. \( \left(\frac{1}{4}\right)^{-4} = 256 \)

Evaluate each expression.
7. \( \log_7 7^3 \)
8. \( \log_{10} 0.001 \)
9. \( \log_8 4096 \)
10. \( \log_4 32 \)
11. \( \log_3 1 \)
12. \( \log_6 \frac{1}{216} \)

Solve each equation.
13. \( \log_x 64 = 3 \)
14. \( \log_4 0.25 = x \)
15. \( \log_4 (2x - 1) = \log_4 16 \)
16. \( \log_{10} \sqrt{10} = x \)
17. \( \log_7 56 - \log_7 x = \log_7 4 \)
18. \( \log_5 (x + 4) + \log_5 x = \log_5 12 \)

19. **Chemistry**  How long would it take 100,000 grams of radioactive iodine, which has a half-life of 60 days, to decay to 25,000 grams? Use the formula \( N = N_0 \left(\frac{1}{2}\right)^t \), where \( N \) is the final amount of the substance, \( N_0 \) is the initial amount, and \( t \) represents the number of half-lives.
Practice

Common Logarithms

Given that $\log 3 = 0.4771$, $\log 5 = 0.6990$, and $\log 9 = 0.9542$, evaluate each logarithm.

1. $\log 300,000$  
2. $\log 0.0005$  
3. $\log 9000$

4. $\log 27$  
5. $\log 75$  
6. $\log 81$

Evaluate each expression.

7. $\log 66.3$  
8. $\log \frac{174}{5}$  
9. $\log 7(4^2)$

Find the value of each logarithm using the change of base formula.

10. $\log_6 832$  
11. $\log_{11} 47$  
12. $\log_3 9$

Solve each equation or inequality.

13. $8^x = 10$  
14. $2.4^x \leq 20$  
15. $1.8^{x-5} = 19.8$

16. $3^{5x} = 85$  
17. $4^{2x} > 25$  
18. $3^{2x-2} = 2^x$

19. Seismology  The intensity of a shock wave from an earthquake is given by the formula $R = \log_{10} \frac{I}{I_0}$, where $R$ is the magnitude, $I$ is a measure of wave energy, and $I_0 = 1$. Find the intensity per unit of area for the following earthquakes.

   a. Northridge, California, in 1994, $R = 6.7$

   b. Hector Mine, California, in 1999, $R = 7.1$
Practice

Natural Logarithms

Evaluate each expression.
1. \( \ln 71 \)
2. \( \ln 8.76 \)
3. \( \ln 0.532 \)

4. \( \text{antiln} -0.256 \)
5. \( \text{antiln} 4.62 \)
6. \( \text{antiln} -1.62 \)

Convert each logarithm to a natural logarithm and evaluate.
7. \( \log_7 94 \)
8. \( \log_5 256 \)
9. \( \log_9 0.712 \)

Use natural logarithms to solve each equation or inequality.
10. \( 6^x = 42 \)
11. \( 7^x = 4^{x+3} \)
12. \( 1249 = 175e^{-0.04t} \)

13. \( 10^{x+1} > 3^x \)
14. \( 12 < e^{0.04xy} \)
15. \( 8.4 < e^{t-2} \)

16. Banking  Ms. Cubbatz invested a sum of money in a certificate of deposit that earns 8% interest compounded continuously. The formula for calculating interest that is compounded continuously is \( A = Pe^r \). If Ms. Cubbatz made the investment on January 1, 1995, and the account was worth $12,000 on January 1, 1999, what was the original amount in the account?
Practice

Modeling Real-World Data with Exponential and Logarithmic Functions

Find the amount of time required for an amount to double at the given rate if the interest is compounded continuously.

1. 4.75%  
2. 6.25%  
3. 5.125%  
4. 7.1%

5. City Planning  At a recent town council meeting, proponents of increased spending claimed that spending should be doubled because the population of the city would double over the next three years. Population statistics for the city indicate that population is growing at the rate of 16.5% per year. Is the claim that the population will double in three years correct? Explain.

6. Conservation  A wildlife conservation group released 14 black bears into a protected area. Their goal is to double the population of black bears every 4 years for the next 12 years.
   a. If they are to meet their goal at the end of the first four years, what should be the yearly rate of increase in population?
   
   b. Suppose the group meets its goal. What will be the minimum number of black bears in the protected area in 12 years?
   
   c. What type of model would best represent such data?
Arithmetic Sequences and Series

Find the next four terms in each arithmetic sequence.
1. $-1.1, 0.6, 2.3, \ldots$
2. $16, 13, 10, \ldots$
3. $p, p+2, p+4, \ldots$

For exercises 4–12, assume that each sequence or series is arithmetic.
4. Find the 24th term in the sequence for which $a_1 = -27$ and $d = 3$.

5. Find $n$ for the sequence for which $a_n = 27$, $a_1 = -12$, and $d = 3$.

6. Find $d$ for the sequence for which $a_1 = -12$ and $a_{23} = 32$.

7. What is the first term in the sequence for which $d = -3$ and $a_6 = 5$?

8. What is the first term in the sequence for which $d = -\frac{1}{3}$ and $a_7 = -3$?

9. Find the 6th term in the sequence $-3 + \sqrt{2}, 0, 3 - \sqrt{2}, \ldots$

10. Find the 45th term in the sequence $-17, -11, -5, \ldots$

11. Write a sequence that has three arithmetic means between 35 and 45.

12. Write a sequence that has two arithmetic means between $7$ and $2.75$.

13. Find the sum of the first 13 terms in the series $-5 + 1 + 7 + \cdots + 67$.

14. Find the sum of the first 62 terms in the series $-23 - 21.5 - 20 - \cdots$

15. Auditorium Design  Wakefield Auditorium has 26 rows, and the first row has 22 seats. The number of seats in each row increases by 4 as you move toward the back of the auditorium. What is the seating capacity of this auditorium?
Practice

Geometric Sequences and Series

Determine the common ratio and find the next three terms of each geometric sequence.

1. \(-1, 2, -4, \ldots\)
2. \(-4, -3, -\frac{9}{4}, \ldots\)
3. \(12, -18, 27, \ldots\)

For exercises 4–9, assume that each sequence or series is geometric.

4. Find the fifth term of the sequence 20, 0.2, 0.002, \ldots.

5. Find the ninth term of the sequence \(\sqrt{3}, -3, 3\sqrt{3}, \ldots\).

6. If \(r = 2\) and \(a_4 = 28\), find the first term of the sequence.

7. Find the first three terms of the sequence for which \(a_4 = 8.4\) and \(r = 4\).

8. Find the first three terms of the sequence for which \(a_6 = \frac{1}{32}\) and \(r = \frac{1}{2}\).

9. Write a sequence that has two geometric means between 2 and 0.25.

10. Write a sequence that has three geometric means between \(-32\) and \(-2\).

11. Find the sum of the first eight terms of the series \(\frac{3}{4} + \frac{9}{20} + \frac{27}{100} + \cdots\).

12. Find the sum of the first 10 terms of the series \(-3 + 12 - 48 + \cdots\).

13. Population Growth  A city of 100,000 people is growing at a rate of 5.2% per year. Assuming this growth rate remains constant, estimate the population of the city 5 years from now.
Practice

Infinite Sequence and Series

Find each limit, or state that the limit does not exist and explain your reasoning.

1. \( \lim_{n \to \infty} \frac{n^2 - 1}{n^2 + 1} \)

2. \( \lim_{n \to \infty} \frac{4n^2 - 5n}{3n^2 + 4} \)

3. \( \lim_{n \to \infty} \frac{5n^2 + 1}{6n} \)

4. \( \lim_{n \to \infty} \frac{(n - 1)(3n + 1)}{5n^2} \)

5. \( \lim_{n \to \infty} \frac{3n - (-1)^n}{4n^2} \)

6. \( \lim_{n \to \infty} \frac{n^3 + 1}{n^2} \)

Write each repeating decimal as a fraction.

7. 0.75

8. 0.592

Find the sum of each infinite series, or state that the sum does not exist and explain your reasoning.

9. \( \frac{2}{5} + \frac{6}{25} + \frac{18}{125} + \ldots \)

10. \( \frac{3}{4} + \frac{15}{8} + \frac{75}{16} + \ldots \)

11. Physics  A tennis ball is dropped from a height of 55 feet and bounces \( \frac{3}{5} \) of the distance after each fall.
   a. Find the first seven terms of the infinite series representing the vertical distances traveled by the ball.
   b. What is the total vertical distance the ball travels before coming to rest?
Use the ratio test to determine whether each series is convergent or divergent.

1. \( \frac{1}{2} + \frac{2^2}{2^2} + \frac{3^2}{2^3} + \frac{4^2}{2^4} + \ldots \)
2. \( 0.006 + 0.06 + 0.6 + \ldots \)
3. \( \frac{4}{1 \cdot 2 \cdot 3} + \frac{8}{1 \cdot 2 \cdot 3 \cdot 4} + \frac{16}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} + \ldots \)
4. \( 5 + \frac{5}{3^3} + \frac{5}{5^3} + \frac{5}{7^3} + \ldots \)

Use the comparison test to determine whether each series is convergent or divergent.

5. \( 2 + \frac{2}{2^3} + \frac{2}{3^3} + \frac{2}{4^3} + \ldots \)
6. \( \frac{5}{2} + 1 + \frac{5}{8} + \frac{5}{11} + \ldots \)

7. Ecology A landfill is leaking a toxic chemical. Six months after the leak was detected, the chemical had spread 1250 meters from the landfill. After one year, the chemical had spread 500 meters more, and by the end of 18 months, it had reached an additional 200 meters.
   a. If this pattern continues, how far will the chemical spread from the landfill after 3 years?
   b. Will the chemical ever reach the grounds of a hospital located 2500 meters away from the landfill? Explain.
Practice

Sigma Notation and the $n$th Term

Write each expression in expanded form and then find the sum.

1. $\sum_{n=3}^{5} (n^2 - 2^n)$

2. $\sum_{q=1}^{4} \frac{2}{q}$

3. $\sum_{t=1}^{5} t(t - 1)$

4. $\sum_{t=0}^{3} (2t - 3)$

5. $\sum_{c=2}^{5} (c - 2)^2$

6. $\sum_{i=1}^{\infty} 10\left(\frac{1}{2}\right)^i$

Express each series using sigma notation.

7. $3 + 6 + 9 + 12 + 15$

8. $6 + 24 + 120 + \cdots + 40,320$

9. $\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{100}$

10. $24 + 19 + 14 + \cdots + (-1)$

11. Savings   Kathryn started saving quarters in a jar. She began by putting two quarters in the jar the first day and then she increased the number of quarters she put in the jar by one additional quarter each successive day.

   a. Use sigma notation to represent the total number of quarters Kathryn had after 30 days.

   b. Find the sum represented in part a.
Practice

The Binomial Theorem

*Use Pascal's triangle to expand each binomial.*

1. \((r + 3)^5\)
2. \((3a - b)^4\)

*Use the Binomial Theorem to expand each binomial.*

3. \((x - 5)^4\)
4. \((3x + 2y)^4\)

5. \((a - \sqrt{2})^5\)
6. \((2p - 3q)^6\)

*Find the designated term of each binomial expansion.*

7. 4th term of \((2n - 3m)^4\)
8. 5th term of \((4a + 2b)^8\)

9. 6th term of \((3p + q)^9\)
10. 3rd term of \((a - 2\sqrt{3})^6\)

11. A varsity volleyball team needs nine members. Of these nine members, at least five must be seniors. How many of the possible groups of juniors and seniors have at least five seniors?
Find each value to four decimal places.
1. \(\ln(-5)\)  
2. \(\ln(-5.7)\)  
3. \(\ln(-1000)\)

Use the first five terms of the exponential series and a calculator to approximate each value to the nearest hundredth.
4. \(e^{0.5}\)  
5. \(e^{1.2}\)  
6. \(e^{2.7}\)  
7. \(e^{0.9}\)

Use the first five terms of the trigonometric series to approximate the value of each function to four decimal places. Then, compare the approximation to the actual value.
8. \(\sin \frac{5\pi}{6}\)  
9. \(\cos \frac{3\pi}{4}\)

Write each complex number in exponential form.
10. \(13 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)\)  
11. \(5 + 5i\)

12. \(1 - \sqrt{3}i\)  
13. \(-7 + 7\sqrt{3}i\)

14. Savings Derika deposited $500 in a savings account with a 4.5% interest rate compounded continuously. (Hint: The formula for continuously compounded interest is \(A = Pe^{rt}\).)

a. Approximate Derika’s savings account balance after 12 years using the first four terms of the exponential series.

b. How long will it take for Derika’s deposit to double, provided she does not deposit any additional funds into her account?
Sequences and Iteration

Find the first four iterates of each function using the given initial value. If necessary, round your answers to the nearest hundredth.

1. \( f(x) = x^2 + 4; \ x_0 = 1 \)
2. \( f(x) = 3x + 5; \ x_0 = -1 \)
3. \( f(x) = x^2 - 2; \ x_0 = -2 \)
4. \( f(x) = x(2.5 - x); \ x_0 = 3 \)

Find the first three iterates of the function \( f(z) = 2z - (3 + i) \) for each initial value.

5. \( z_0 = i \)
6. \( z_0 = 3 - i \)
7. \( z_0 = 0.5 + i \)
8. \( z_0 = -2 - 5i \)

Find the first three iterates of the function \( f(z) = z^2 + c \) for each given value of \( c \) and each initial value.

9. \( c = 1 - 2i; \ z_0 = 0 \)
10. \( c = i; \ z_0 = i \)
11. \( c = 1 + i; \ z_0 = -1 \)
12. \( c = 2 - 3i; \ z_0 = 1 + i \)

13. **Banking** Mai deposited $1000 in a savings account. The annual yield on the account is 5.2%. Find the balance of Mai’s account after each of the first 3 years.
Use mathematical induction to prove that each proposition is valid for all positive integral values of $n$.

1. \[ \frac{1}{3} + \frac{2}{3} + \frac{3}{3} + \ldots + \frac{n}{3} = \frac{n(n + 1)}{6} \]

2. $5^n + 3$ is divisible by 4.
Permutations and Combinations

1. A golf manufacturer makes irons with 7 different shaft lengths, 3 different grips, and 2 different club head materials. How many different combinations are offered?

2. A briefcase lock has 3 rotating cylinders, each containing 10 digits. How many numerical codes are possible?

3. How many 7-digit telephone numbers can be formed if the first digit cannot be 0 or 1?

Find each value.

4. \(P(10, 7)\) 
5. \(P(7, 7)\) 
6. \(P(6, 3)\) 

7. \(C(7, 2)\) 
8. \(C(10, 4)\) 
9. \(C(12, 4) \cdot C(8, 3)\) 

10. How many ways can the 4 call letters of a radio station be arranged if the first letter must be W or K and no letters can be repeated?

11. There are 5 different routes that a commuter can take from her home to her office. How many ways can she make a roundtrip if she uses different routes for coming and going?

12. How many committees of 5 students can be selected from a class of 25?

13. A box contains 12 black and 8 green marbles. How many ways can 3 black and 2 green marbles be chosen?

14. **Basketball** How many ways can a coach select a starting team of one center, two forwards, and two guards if the basketball team consists of three centers, five forwards, and three guards?
Practice

Permutations with Repetitions and Circular Permutations

How many different ways can the letters of each word be arranged?

1. members 2. annually

3. Missouri 4. concert

5. How many different 5-digit street addresses can have the digits 4, 7, 3, 4, and 8?

6. Three hardcover books and 5 paperbacks are placed on a shelf. How many ways can the books be arranged if all the hardcover books must be together and all the paperbacks must be together?

Determine whether each arrangement of objects is a linear or circular permutation. Then determine the number of arrangements for each situation.

7. 9 keys on a key ring with no chain

8. 5 charms on a bracelet with no clasp

9. 6 people seated at a round table with one person seated next to a door

10. 12 different symbols around the face of a watch

11. Entertainment Jasper is playing a word game and has the following letters in his tray: QUOUNNTAGGRA. How many 12-letter arrangements could Jasper make to check if a single word could be formed from all the letters?
Probability and Odds

A kitchen drawer contains 7 forks, 4 spoons, and 5 knives. Three are selected at random. Find each probability.

1. \( P(3 \text{ forks}) \)
2. \( P(2 \text{ forks, 1 knife}) \)
3. \( P(3 \text{ spoons}) \)
4. \( P(1 \text{ fork, 1 knife, 1 spoon}) \)

A laundry bag contains 5 red, 9 blue, and 6 white socks. Two socks are selected at random. Find each probability.

5. \( P(2 \text{ red}) \)
6. \( P(2 \text{ blue}) \)
7. \( P(1 \text{ red, 1 blue}) \)
8. \( P(1 \text{ red, 1 white}) \)

Sharon has 8 mystery books and 9 science-fiction books. Four are selected at random. Find each probability.

9. \( P(4 \text{ mystery books}) \)
10. \( P(4 \text{ science-fiction books}) \)
11. \( P(2 \text{ mysteries, 2 science-fiction}) \)
12. \( P(3 \text{ mysteries, 1 science-fiction}) \)

From a standard deck of 52 cards, 5 cards are drawn. What are the odds of each event occurring?

13. 5 aces
14. 5 face cards

15. Meteorology  A local weather forecast states that the chance of sunny weather on Wednesday is 70%. What are the odds that it will be sunny on Wednesday?
Practice

Probabilities of Compound Events

**Determine if each event is independent or dependent. Then determine the probability.**

1. the probability of drawing a black card from a standard deck of cards, replacing it, then drawing another black card

2. the probability of selecting 1 jazz, 1 country, and 1 rap CD in any order from 3 jazz, 2 country, and 5 rap CDs, replacing the CDs each time

3. the probability that two cards drawn from a deck are both aces

**Determine if each event is mutually exclusive or mutually inclusive. Then determine each probability.**

4. the probability of rolling a 3 or a 6 on one toss of a number cube

5. the probability of selecting a queen or a red card from a standard deck of cards

6. the probability of selecting at least three white crayons when four crayons are selected from a box containing 7 white crayons and 5 blue crayons

7. **Team Sports** Conrad tried out for both the volleyball team and the football team. The probability of his being selected for the volleyball team is \( \frac{4}{5} \), while the probability of his being selected for the football team is \( \frac{3}{4} \). The probability of his being selected for both teams is \( \frac{7}{10} \). What is the probability that Conrad will be selected for either the volleyball team or the football team?
Practice

Conditional Probabilities

Find each probability.

1. Two number cubes are tossed. Find the probability that the numbers showing on the cubes match, given that their sum is greater than 7.

2. A four-digit number is formed from the digits 1, 2, 3, and 4. Find the probability that the number ends in the digits 41, given that the number is odd.

3. Three coins are tossed. Find the probability that exactly two coins show tails, given that the third coin shows tails.

A card is chosen from a standard deck of cards. Find each probability, given that the card is red.

4. \(P(\text{diamond})\)

5. \(P(\text{six of hearts})\)

6. \(P(\text{queen or 10})\)

7. \(P(\text{face card})\)

A survey taken at Stirers High School shows that 48% of the respondents like soccer, 66% like basketball, and 38% like hockey. Also, 30% like soccer and basketball, 22% like basketball and hockey, and 28% like soccer and hockey. Finally, 12% like all three sports.

8. If Meg likes basketball, what is the probability that she also likes soccer?

9. If Jaime likes soccer, what is the probability that he also likes hockey and basketball?

10. If Ashley likes basketball, what is the probability that she also likes hockey?

11. If Brett likes soccer, what is the probability that he also likes basketball?
The Binomial Theorem and Probability

Find each probability if six coins are tossed.

1. \( P(3 \text{ heads and 3 tails}) \)
2. \( P(\text{at least 4 heads}) \)

3. \( P(2 \text{ heads or 3 tails}) \)
4. \( P(\text{all heads or all tails}) \)

The probability of Chris’s making a free throw is \( \frac{2}{3} \). Find each probability if she shoots five times.

5. \( P(\text{all missed}) \)
6. \( P(\text{all made}) \)

7. \( P(\text{exactly 4 made}) \)
8. \( P(\text{at least 3 made}) \)

When Maria and Len play a certain board game, the probability that Maria will win the game is \( \frac{3}{4} \). Find each probability if they play five games.

9. \( P(\text{Len wins only 1 game}) \)
10. \( P(\text{Maria wins exactly 2 games}) \)

11. \( P(\text{Len wins at least 2 games}) \)
12. \( P(\text{Maria wins at least 3 games}) \)

13. Gardening  Assume that 60% of marigold seeds that are sown directly in the ground produce plants. If Tomaso plants 10 seeds, what is the probability that 7 plants will be produced?
The Frequency Distribution

Determine which class intervals would be appropriate for the data below. Explain your answers.

1. 25, 32, 18, 99, 43, 16, 29, 35, 34, 10, 21, 26, 26, 17, 40, 22, 38, 16, 19
   a. 1
   b. 10
   c. 2

2. 111, 115, 130, 200, 234, 98, 115, 72, 305, 145, 87, 63, 245, 285, 256, 302
   a. 25
   b. 10
   c. 30

3. Meteorology  The average wind speeds recorded at various weather stations in the United States are listed below.

<table>
<thead>
<tr>
<th>Station</th>
<th>Speed (mph)</th>
<th>Station</th>
<th>Speed (mph)</th>
<th>Station</th>
<th>Speed (mph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albuquerque</td>
<td>8.9</td>
<td>Anchorage</td>
<td>7.1</td>
<td>Atlanta</td>
<td>9.1</td>
</tr>
<tr>
<td>Baltimore</td>
<td>9.1</td>
<td>Boston</td>
<td>12.5</td>
<td>Chicago</td>
<td>10.4</td>
</tr>
<tr>
<td>Dallas-Ft. Worth</td>
<td>10.8</td>
<td>Honolulu</td>
<td>11.3</td>
<td>Indianapolis</td>
<td>9.6</td>
</tr>
<tr>
<td>Kansas City</td>
<td>10.7</td>
<td>Las Vegas</td>
<td>9.3</td>
<td>Little Rock</td>
<td>7.8</td>
</tr>
<tr>
<td>Los Angeles</td>
<td>6.2</td>
<td>Memphis</td>
<td>8.8</td>
<td>Miami</td>
<td>9.2</td>
</tr>
<tr>
<td>Minneapolis– St. Paul</td>
<td>10.5</td>
<td>New Orleans</td>
<td>8.1</td>
<td>New York City</td>
<td>9.4</td>
</tr>
<tr>
<td>Philadelphia</td>
<td>9.5</td>
<td>Phoenix</td>
<td>6.2</td>
<td>Seattle</td>
<td>9.0</td>
</tr>
</tbody>
</table>

Source: National Climatic Data Center

a. Find the range of the data.

b. Determine an appropriate class interval.

c. What are the class limits and the class marks?

d. Construct a frequency distribution of the data.

e. Draw a histogram of the data.
Measures of Central Tendency

*Find the mean, median, and mode of each set of data.*

1. \{15, 42, 26, 39, 93, 42\}

2. \{32, 12, 61, 94, 73, 62, 94, 35, 44, 52\}

3. \{152, 697, 202, 312, 109, 134, 116\}

4. \{18, 6, 22, 33, 19, 34, 14, 54, 12, 22, 19\}

5. A shoe store employee sets up a display by placing shoeboxes in 10 stacks. The numbers of boxes in each stack are 5, 7, 9, 11, 13, 10, 9, 8, 7, and 5.
   a. What is the mean of the number of boxes in a stack?
   b. Find the median of the number of boxes in a stack.
   c. If one box is removed from each stack, how will the mean and median be affected?

*Find the mean, median, and mode of the data represented by each stem-and-leaf plot.*

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
<th>Stem</th>
<th>Leaf</th>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2 4 4 7</td>
<td>9</td>
<td>0 1 1 3</td>
<td>1</td>
<td>1 2 9</td>
</tr>
<tr>
<td>3</td>
<td>1 3 4</td>
<td>10</td>
<td>1 3 5 6</td>
<td>2</td>
<td>3 3 5</td>
</tr>
<tr>
<td>4</td>
<td>5 6 8</td>
<td>11</td>
<td>3 4 6 8</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>9 9 0 = 90</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

5. 2 2 = 220

8.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4 5 6 8 9</td>
</tr>
<tr>
<td>1/1</td>
<td>1.1</td>
</tr>
</tbody>
</table>

9. **Medicine** A frequency distribution for the number of patients treated at 50 U.S. cancer centers in one year is given at the right.
   a. Use the frequency chart to find the mean of the number of patients treated by a cancer center.
   b. What is the median class of the frequency distribution?

<table>
<thead>
<tr>
<th>Patients</th>
<th>Number of Cancer Centers</th>
</tr>
</thead>
<tbody>
<tr>
<td>500–1000</td>
<td>26</td>
</tr>
<tr>
<td>1000–1500</td>
<td>14</td>
</tr>
<tr>
<td>1500–2000</td>
<td>6</td>
</tr>
<tr>
<td>2000–2500</td>
<td>0</td>
</tr>
<tr>
<td>2500–3000</td>
<td>2</td>
</tr>
<tr>
<td>3000–3500</td>
<td>0</td>
</tr>
<tr>
<td>3500–4000</td>
<td>2</td>
</tr>
</tbody>
</table>

*Source: U.S. News Online*
Practice

Measures of Variability

*Find the interquartile range and the semi-interquartile range of each set of data. Then draw a box-and-whisker plot.*

1. 43, 26, 92, 11, 8, 49, 52, 126, 86, 42, 63, 78, 91, 79, 86

2. 1.6, 9.8, 4.5, 6.2, 8.7, 5.6, 3.9, 6.8, 9.7, 1.1, 4.7, 3.8, 7.5, 2.8, 0.1

*Find the mean deviation and the standard deviation of each set of data.*

3. 146, 289, 121, 146, 212, 98, 86, 153, 128, 136, 181, 142

4. 1592, 1486, 1479, 1682, 1720, 1104, 1486, 1895, 1890, 2687, 2450

5. *Sociology*  The frequency distribution at the right shows the average life expectancy for males and females in 15 European Union countries in 1994.
   a. Find the mean of the female life expectancy.
   b. Find the mean of the male life expectancy.
   c. What is the standard deviation of the female life expectancy?
   d. What is the standard deviation of the male life expectancy?

<table>
<thead>
<tr>
<th>Life Expectancy (years)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>71.5–73.0</td>
<td>3</td>
</tr>
<tr>
<td>73.0–74.5</td>
<td>9</td>
</tr>
<tr>
<td>74.5–76.0</td>
<td>2</td>
</tr>
<tr>
<td>76.0–77.5</td>
<td>1</td>
</tr>
<tr>
<td>77.5–79.0</td>
<td>0</td>
</tr>
<tr>
<td>79.0–80.5</td>
<td>8</td>
</tr>
<tr>
<td>80.5–82.0</td>
<td>4</td>
</tr>
</tbody>
</table>

   *Source: Department of Health and Children, Ireland*
Practice

The Normal Distribution

A set of 1000 values has a normal distribution. The mean of the data is 120, and the standard deviation is 20.

1. How many values are within one standard deviation of the mean?

2. What percent of the data is between 110 and 130?

3. What percent of the data is between 90 and 110?

4. Find the interval about the mean that includes 90% of the data.

5. Find the interval about the mean that includes 77% of the data.

6. Find the limit below which 90% of the data lie.

7. Dog Breeding  The weights of full-grown German shepherds at the City View Kennels are normally distributed. The mean weight is 86 pounds, and the standard deviation is 3 pounds. Skipper, a full-grown German shepard, weighs 79 pounds.
   a. What percent of the full-grown German shepherds at City View Kennels weigh more than Skipper?
   b. What percent of the full-grown German shepherds at City View Kennels weigh less than Skipper?
Sample Sets of Data

Find the standard error of the mean for each sample. Then find the interval about the sample mean that has a 1% level of confidence and the interval about the sample mean that has a 5% level of confidence.

1. \( \sigma = 50, N = 100, \bar{X} = 250 \)
2. \( \sigma = 4, N = 64, \bar{X} = 100 \)

3. \( \sigma = 2.6, N = 250, \bar{X} = 50 \)
4. \( \sigma = 4.3, N = 375, \bar{X} = 110 \)

The table below shows a frequency distribution of the time in minutes required for students to wash a car during a car wash fundraiser. The distribution is a random sample of 250 cars. Use the table for Exercises 5-10.

<table>
<thead>
<tr>
<th>Number of Minutes</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>1</td>
<td>8</td>
<td>5</td>
</tr>
</tbody>
</table>

5. What is the mean of the data in the frequency distribution?

6. Find the standard deviation of the data.

7. Find the standard error of the mean.

8. Find the interval about the sample mean such that the probability is 0.90 that the true mean lies within the interval.

9. Find the interval about the sample mean such that the probability is 0.95 that the true mean lies within the interval.

10. Determine the interval about the sample mean that has a 1% level of confidence.
Limits

Evaluate each limit.

1. \( \lim_{x \to 3} (x^2 + 3x - 8) \)

2. \( \lim_{x \to 2} (2x + 7) \)

3. \( \lim_{x \to 6} \frac{x^2 - 36}{x + 6} \)

4. \( \lim_{x \to 2} \frac{x^2 - 5x + 6}{x - 2} \)

5. \( \lim_{x \to 2} \frac{x - 2}{x^2 - 4} \)

6. \( \lim_{x \to 3} \frac{x^2 - 9}{x^3 - 27} \)

7. \( \lim_{x \to 0} \frac{(3 + x)^2 - 9}{x} \)

8. \( \lim_{x \to 4} \sqrt{x^2 - 2x + 1} \)

9. \( \lim_{x \to 3} \frac{2x^3 + 6x^2 - x - 3}{x + 3} \)

10. \( \lim_{x \to 1} \frac{x^2 - x}{2x^2 + 5x - 7} \)

11. \( \lim_{x \to 0} \frac{\sin 2x}{x} \)

12. \( \lim_{x \to 0} \frac{1 - \cos x}{x} \)

13. Biology The number of cells in a culture doubles every 5 hours. The initial hourly growth rate is represented by

\[ \lim_{t \to 0} \left( 2^{\frac{t}{5}} - 1 \right) \], where \( t \) is the time in hours. Use a calculator to approximate the value of this limit to the nearest hundredth. What is the initial hourly growth rate? Write your answer as a percent.
Practice

Derivatives and Antiderivatives

Use the derivative rules to find the derivative of each function.

1. \( f(x) = 2x^2 - 3x \)  
2. \( f(x) = 6x^3 - 2x + 5 \)

3. \( f(x) = (2x + 7)(3x - 8) \)  
4. \( f(x) = (x^2 + 1)(3x - 2) \)

5. \( f(x) = (x^2 + 5x)^2 \)  
6. \( f(x) = x^2(x^3 + 3x^2) \)

Find the antiderivative of each function.

7. \( f(x) = x^8 \)  
8. \( f(x) = 4x^3 \)

9. \( f(x) = 2x + 3 \)  
10. \( f(x) = x(x^2 - 3) \)

11. \( f(x) = (2x + 1)(3x - 2) \)  
12. \( f(x) = 8x^2 + 2x - 3 \)

13. **Physics** Acceleration is the rate at which the velocity of a moving object changes. The velocity in meters per second of a particle moving along a straight line is given by the function \( v(t) = 3t^2 - 6t + 5 \), where \( t \) is the time in seconds. Find the acceleration of the particle in meters per second squared after 5 seconds. (Hint: Acceleration is the derivative of velocity.)
Area Under a Curve

Use limits to find the area between each curve and the x-axis for the given interval.

1. $y = x^3$ from $x = 0$ to $x = 2$
2. $y = x^2$ from $x = 1$ to $x = 6$

Use limits to evaluate each integral.

3. $\int_1^5 x^3$
4. $\int_1^3 x^4$

5. Architecture and Design  A designer is making a stained-glass window for a new building. The shape of the window can be modeled by the parabola $y = 5 - 0.05x^2$. What is the area of the window?

NAME ___________________________   DATE _______________   PERIOD ________
Practice

The Fundamental Theorem of Calculus

Evaluate each indefinite integral.
1. \( \int 8 \, dx \)
2. \( \int (2x^3 + 6x) \, dx \)
3. \( \int (-6x^5 - 2x^2 + 5x) \, dx \)
4. \( \int (9x^2 + 12x - 9) \, dx \)

Evaluate each definite integral.
5. \( \int_{2}^{5} 2x \, dx \)
6. \( \int_{-5}^{-1} (-4x^3 - 3x^2) \, dx \)
7. \( \int_{-2}^{1} (1 - x)(x + 3) \, dx \)
8. \( \int_{-1}^{2} (x + 1)^3 \, dx \)

Find the area between each curve and the x-axis for the given interval.
9. \( y = x^2 + 4x + 5 \) from \( x = 1 \) to \( x = 4 \)
10. \( y = x + 1 \) from \( x = -1 \) to \( x = 2 \)
11. \( y = x^3 + 1 \) from \( x = -1 \) to \( x = 3 \)
12. \( y = x^2 + 1 \) from \( x = 0 \) to \( x = 3 \)

13. Physics  The work in foot-pounds required to compress a certain spring a distance of \( \ell \) feet from its natural length is given by \( W = \int_{0}^{\ell} 2x \, dx \). How much work is required to compress the spring 6 inches from its natural length?